# Homework 3

Deadline: Wednesday, December 11 at 14:00.

*Please submit your solutions either on paper at the beginning of the practicals or as a pdf-file in the SIS using the Study group roster (Studijní mezivýsledky) application. A maximum of 5 points can be awarded for each task. The solution to each problem must be explained. Everything that is not immediately obvious needs to be proved or quoted from lecture notes.*

1. Find $\gcd(4+3i, 3+i)$ in the domain of Gaussian integers $\mathbb{Z}[i]$.

2. Prove that the subdomain $\mathbb{Z}[\sqrt{5}]$ of the field of complex numbers is not Euclidean.

3. Calculate all possible values of $\gcd(36x^3 - 42x^2 + 6, 36x^3 - 24x^2 - 48x - 12)$ in $\mathbb{Z}[x]$.

4. Prove that $\mathbb{Z}_2[\alpha]/(\alpha^4 + \alpha^3 + 1)$ forms a field, determine the number of its elements, and calculate $\alpha^{-1}$ and $\alpha^9$ in the field.

5. Find a polynomial $f \in \mathbb{Z}_7[x]$ of the smallest possible degree satisfying all the conditions $f \equiv 2x + 5 \pmod{x^2 + 2}$, $f \equiv 1 \pmod{x}$, $f \equiv 1 \pmod{x + 1}$.