CVIČENÍ Z MA 2, 22.5.2020

(rovnice se separovanými proměnnými)

$y' = g(y)h(x)$

$y : (a,b) \longrightarrow \mathbb{R} \qquad \forall x \in (a,b) : y'(x) = g(y(x)) \cdot h(x)$

$G' = \frac{1}{g} \qquad H' = h$

$\boxed{1}$ $y' = \sqrt[5]{y^2}$

$g(y) = \sqrt[5]{y^2}, \ y \in \mathbb{R} \ ; \ h(x) = 1, \ x \in \mathbb{R}$

$g(c) = 0 \iff c = 0$

singulární řešení: $y(x) = 0, \ x \in \mathbb{R}$

(a) $y \in (0, \infty)$

$\int \frac{1}{\sqrt[5]{y^2}} dy \overset{c}{=} \frac{5}{3} y^{\frac{3}{5}}, \ y \in (0, \infty) \qquad G(y) = \frac{5}{3} y^{\frac{3}{5}}$

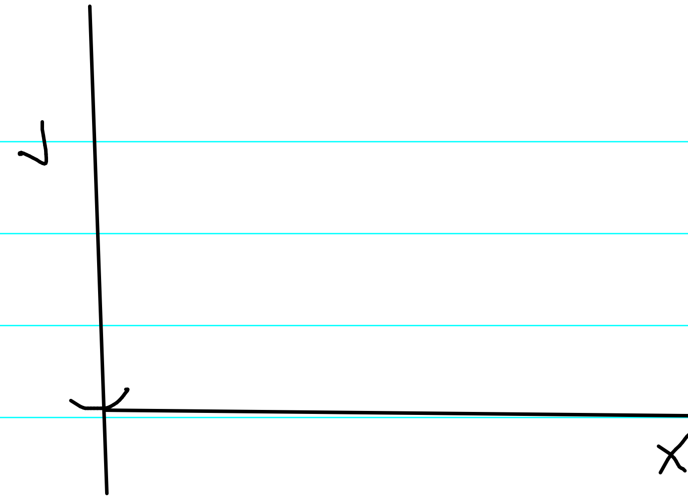
$\int 1 dx \overset{c}{=} x, \ x \in \mathbb{R} \qquad H(x) = x$

$$G(y) = H(x) + c$$

$$G(y(x)) = H(x) + c$$

$$y(x) = G^{-1}(\underbrace{H(x)+c}_{\in D(G^{-1})})$$

$$D(G^{-1}) = G(J)$$



$$H(x)+c \in G(J)$$

$$\frac{5}{3} y^{3/5} = x + c$$

$$x + c \in G((0,\infty)) = (0,\infty)$$

$$x \in (-c, \infty)$$

$$y(x) = \left(\frac{3}{5}(x+c)\right)^{5/3}, \quad x \in (-c,\infty), \quad c \in \mathbb{R}$$

(b) $y \in (-\infty, 0)$

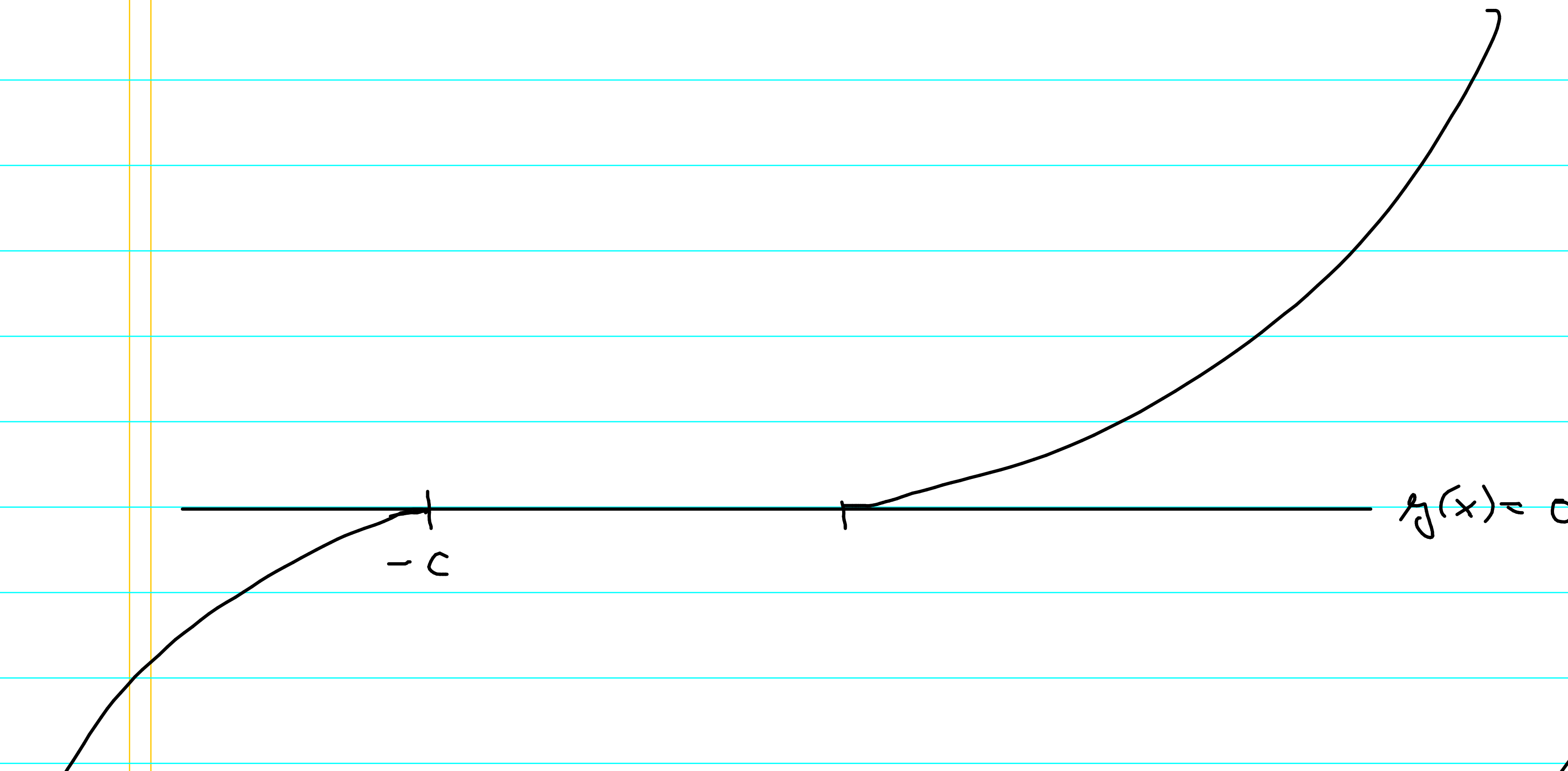
$$G(y) = \frac{5}{3}\sqrt[5]{y^3} \qquad G((-\infty,0)) = (-\infty,0)$$

$$x + c \in G((-\infty,0))$$

$$x + c < 0$$

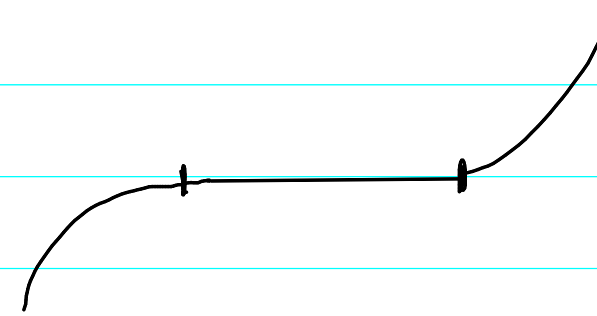
$$x < -c$$

$$y(x) = \sqrt[3]{\left[\frac{3}{5}(x+c)\right]^5}, \quad x \in (-\infty,-c), \quad c \in \mathbb{R}$$

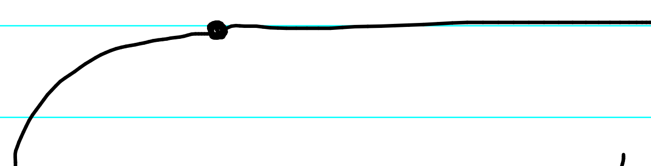


(1)

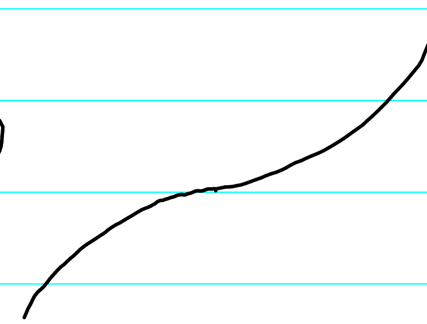
(4)



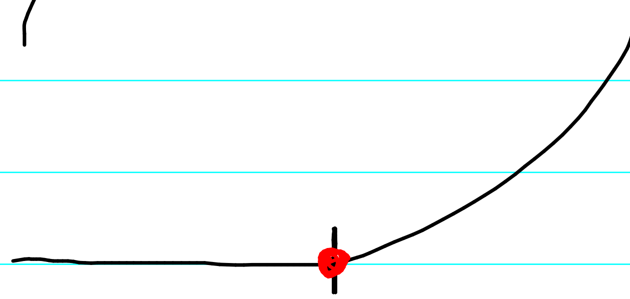
(2)



(5)



(3)



ad (4)

$$y(x) = \begin{cases} \sqrt[3]{\left(\frac{3}{5}(x+c_1)\right)^5}, & x \in (-\infty, -c_1\rangle \\ 0, & x \in \langle -c_1, -c_2\rangle \\ \sqrt[3]{\left(\frac{3}{5}(x+c_2)\right)^5}, & x \in (-c_2, \infty) \end{cases} \qquad c_2 < c_1$$

$\boxed{2}$ $y' = \frac{y^2}{x^2}$

- všechna maximální řešení
- maximální řešení procházející bodem $[1, \frac{1}{2}]$ $(y(1) = \frac{1}{2})$
- všechna maximální řešení, která jsou omezená

$h(x) = \frac{1}{x^2}$ $\quad I = (-\infty, 0)$ nebo $(0, \infty)$

$g(y) = y^2$ $\quad g > 0$ na $(-\infty, 0) \cup (0, \infty)$

$g(y) = 0 \iff y = 0$

singulární řešení: $y(x) = 0,\ x \in (-\infty, 0)$

$y(x) = 0,\ x \in (0, \infty)$

(a) $x \in (0, \infty),\ y \in (0, \infty)$

$\int \frac{1}{x^2} dx \overset{c}{=} -\frac{1}{x},\ x \in (0, \infty)$ $\quad \int \frac{1}{y^2} dy \overset{c}{=} -\frac{1}{y},\ y \in (0, \infty)$

$G(y) = H(x) + c$

$\{x \in (0, \infty);\ H(x) + c \in G((0, \infty))\} = \{x \in (0, \infty);\ -\frac{1}{x} + c < 0\}$

$= \begin{cases} (0, \infty), & c \leq 0 \\ (0, \frac{1}{c}) & c > 0 \end{cases}$

$-\frac{1}{y} = -\frac{1}{x} + c \implies y(x) = \frac{x}{1 - cx}$

(b) $x \in (0,\infty)$, $y \in (-\infty, 0)$

$$\left\{ x \in (0,\infty);\ -\frac{1}{x} + c > 0 \right\} = \begin{cases} \emptyset, & c \le 0 \\ \left(\frac{1}{c}, \infty\right) & c > 0 \end{cases}$$

$$y(x) = \frac{x}{1 - cx}, \quad x \in \left(\frac{1}{c}, \infty\right), \quad c > 0$$

(c) $x \in (-\infty, 0)$, $y \in (0, \infty)$

$$\left\{ x \in (-\infty, 0);\ -\frac{1}{x} + c < 0 \right\} = \begin{cases} \emptyset, & c \ge 0 \\ \left(-\infty, \frac{1}{c}\right) & c < 0 \end{cases}$$

$$y(x) = \frac{x}{1 - cx}, \quad x \in \left(-\infty, \frac{1}{c}\right), \quad c < 0$$

(d) $x \in (-\infty, 0)$, $y \in (-\infty, 0)$

$$\left\{ x \in (-\infty, 0);\ -\frac{1}{x} + c > 0 \right\} = \begin{matrix} (-\infty, 0) & c \ge 0 \\ \left(\frac{1}{c}, 0\right) & c < 0 \end{matrix}$$

$$y(x) = \frac{x}{1 - cx}$$

$$y(1) = \frac{1}{2} \qquad y(x) = \frac{x}{1 - cx} \qquad \underline{y(x) = \frac{x}{1 + x}}$$

$$\frac{1}{1 - c} = \frac{1}{2} \Rightarrow c = -1$$

omezená maximální řešení

$y(x) = 0, \quad x \in (-\infty, 0)$

$y(x) = 0, \quad x \in (0, \infty)$

$$y(x) = \frac{x}{1 - cx} \qquad x \in (-\infty, 0),\ c > 0$$

$$x \in (0, \infty),\ c < 0$$

$\frac{x}{1-cx}$	$(0, \infty)$	$c \leq 0$	
	$(0, \frac{1}{c})$	$c > 0$	$\times$
	$(\frac{1}{c}, \infty)$	$c > 0$	$\times$