CVIČENÍ 2 MA 2, 13.5.2020

Limitní srovnávací kritérium: Nechť $a, b \in \mathbb{R}^*$, $a < b$, f, g jsou spojité nezáporné na $[a, b)$ a $\lim\limits_{x \to b-} f(x)/g(x) = c \in (0, \infty)$.
Potom $f \in \mathcal{N}(a, b) \Leftrightarrow g \in \mathcal{N}(a, b)$.

$\boxed{1}$ $$\int_0^1 \underbrace{\frac{\operatorname{tg} x}{\sqrt{x^3}}}_{f} \, dx$$

$$\frac{\operatorname{tg} x}{x \cdot \sqrt{x}} = \underbrace{\frac{\operatorname{tg} x}{x}}_{\to 1 \quad x \to 0+} \cdot \frac{1}{\sqrt{x}}$$

$g(x) = \frac{1}{\sqrt{x}}$, $x \in (0, 1]$ $\qquad \lim\limits_{x \to 0+} \frac{f(x)}{g(x)} = 1 \in (0, \infty)$

$f \in \mathcal{N}(0,1) \Leftrightarrow g \in \mathcal{N}(0,1)$ $\qquad \int_0^1 x^{-\frac{1}{2}} dx$ K

$\int_0^1 \frac{\operatorname{tg} x}{\sqrt{x^3}} dx$ K

$\boxed{2}$ $\displaystyle\int_0^{\pi/2} \underbrace{(\operatorname{tg} x)^\alpha}_{f(x)} dx, \ \alpha \in \mathbb{R}$

VÝSLEDEK:

$$\int_0^{\pi/2} (\operatorname{tg} x)^\alpha dx \ K \iff \alpha \in (-1,1)$$

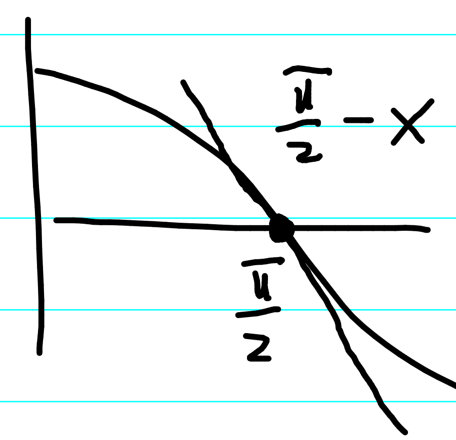
bod 0:

$$\int_0^1 (\operatorname{tg} x)^\alpha dx \ K \iff \int_0^1 x^\alpha dx \ K \iff \alpha > -1$$

$$g(x) = x^\alpha \qquad \lim_{x\to 0+} \frac{f(x)}{g(x)} = \lim_{x\to 0+} \underbrace{\left(\frac{\operatorname{tg} x}{x}\right)}^\alpha = 1 \in (0,\infty)$$

bod $\frac{\pi}{2}$:

$$\int_1^{\pi/2} f(x)\,dx$$



$$\operatorname{tg} x = \frac{\sin x}{\cos x} \qquad \sin x \to 1, \quad \cos x \to 0$$

$$\lim_{x\to\frac{\pi}{2}} \frac{\cos x}{\frac{\pi}{2} - x} = \lim_{x\to\frac{\pi}{2}} \frac{-\sin x}{-1} = 1$$

$$\lim_{x\to\frac{\pi}{2}-} \frac{\operatorname{tg} x}{\frac{1}{\frac{\pi}{2}-x}} = \lim_{x\to\frac{\pi}{2}-} \sin x \cdot \frac{\frac{\pi}{2}-x}{\cos x} = 1$$

$-\alpha > -1$

$1 > \alpha$

$$\frac{\pi}{2} - x = y$$

$$g(x) = \frac{1}{(\frac{\pi}{2}-x)^\alpha} \qquad \int_1^{\pi/2} \frac{1}{(\frac{\pi}{2}-x)^\alpha} dx = \int_{\frac{\pi}{2}-1}^{0} \frac{1}{y^\alpha}(-1)\,dy = \int_0^{\frac{\pi}{2}-1} y^{-\alpha}\,dy$$

$\boxed{3}$ $$\int_{-1}^{1} \underbrace{\frac{1}{\sqrt{1-x^4}}}_{f(x)}\, dx$$

$$1-x^4 = (1-x^2)(1+x^2) = (1-x)(1+x)(1+x^2)$$

$$f(x) = \frac{1}{\sqrt{1+x^2}} \cdot \frac{1}{\sqrt{1-x}} \cdot \frac{1}{\sqrt{1+x}}$$

<u>bod −1:</u> $\int_{-1}^{0} f(x)\,dx$ K

$g(x) = \frac{1}{\sqrt{1+x}}$ $\qquad \lim_{x \to -1_+} \frac{f(x)}{g(x)} = \frac{1}{\sqrt{2}} \cdot \frac{1}{\sqrt{2}} \in (0, \infty)$

$\int_{-1}^{0} f$ K $\Leftrightarrow \int_{-1}^{0} g$ $\qquad \int_{-1}^{0} \frac{1}{\sqrt{1+x}}\,dx = \int_{0}^{1} \frac{1}{\sqrt{y}}\,dy$ K

$1+x = y$

<u>bod 1:</u> $\int_{0}^{1} f$ K $\qquad g(x) = \frac{1}{\sqrt{1-x}}$ $\qquad \lim_{x \to 1_-} \frac{f(x)}{g(x)} = \frac{1}{2} \in (0, \infty)$

$\int_{0}^{1} \frac{1}{\sqrt{1-x}}\,dx$ K $\qquad 1-x = y$

$\int_{-1}^{1} \frac{1}{\sqrt{1-x^4}}\,dx$ konverguje

$\boxed{4}$ $\displaystyle\int_0^\infty \frac{\sin x}{x}\,dx$ K

$\displaystyle\int_0^1 \frac{\sin x}{x}\,dx$ K $\qquad \displaystyle\int_1^\infty \frac{\sin x}{x}\,dx$

$\displaystyle\int_0^1 \frac{1}{s}\sin\frac{1}{s}\,ds$ $\qquad s \in (0,\infty)$

$$\int \cos\frac{1}{s}\,ds = \int 1\cdot\cos\frac{1}{s}\,ds = s\cdot\cos\frac{1}{s} - \int s\left(-\sin\frac{1}{s}\cdot\frac{-1}{s^2}\right)ds$$

$$= s\cdot\cos\frac{1}{s} - \int \frac{1}{s}\sin\frac{1}{s}\,ds$$

$$\int_0^1 \frac{1}{s}\sin\frac{1}{s}\,ds = \underset{\in\mathbb{R}}{\left[s\cdot\cos\frac{1}{s}\right]_0^1} - \underbrace{\int_0^1 \cos\frac{1}{s}\,ds}_{\in\mathbb{R}}$$

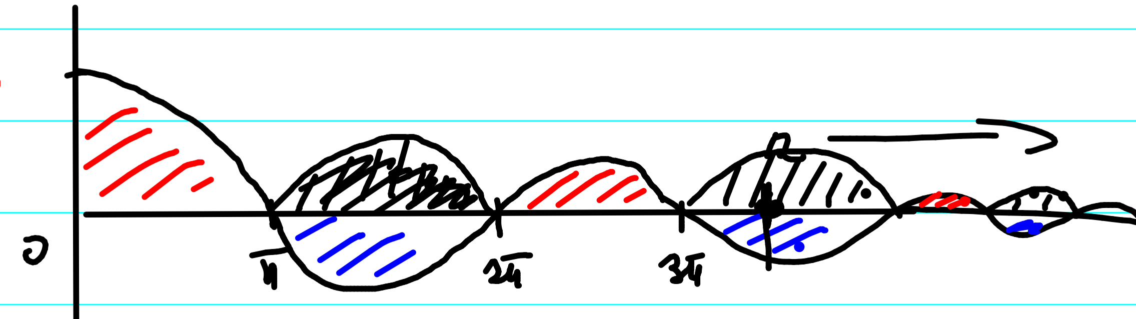
$$\int_1^\infty \frac{\sin x}{x}\,dx = \int_1^0 \frac{\sin\frac{1}{s}}{\frac{1}{s}}\cdot\frac{-1}{s^2}\,ds = \int_0^1 \frac{1}{s}\cdot\sin\frac{1}{s}\,ds \in \mathbb{R}$$

$x = \frac{1}{s}$

$dx = -\frac{1}{s^2}\,ds$

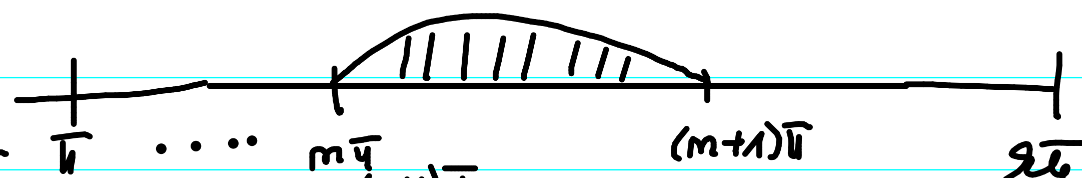
$\boxed{5}$ $\int_0^\infty \overbrace{\frac{|\sin x|}{x}}^{f(x)} dx$ D

$x \mapsto \frac{\sin x}{x} = \frac{1}{x} \cdot \sin x$



$\lim_{n \to \infty} \int_0^n f$

$\int_0^\infty f$ K $\quad \int_0^\infty |f|$ D



$$\int_{m\pi}^{(m+1)\pi} f(x)dx = \int_{m\pi}^{(m+1)\pi} \frac{1}{x} |\sin x| dx \geq \int_{m\pi}^{(m+1)\pi} \frac{1}{(m+1)\pi} \cdot |\sin x| dx$$

$$= \frac{1}{(m+1)\pi} \int_0^\pi \sin x \, dx = \frac{2}{(m+1)\pi}$$

$$\int_\pi^\infty |f| \geq \int_\pi^{(k+1)\pi} |f| = \sum_{m=1}^{k} \int_{m\pi}^{(m+1)\pi} |f| \geq \sum_{m=1}^{k} \frac{2}{(m+1)\pi} \xrightarrow[k \to \infty]{} \infty$$

$\int_\pi^\infty |f| = \infty \qquad \int_0^\infty |f| = \infty$