Introductory Mathematics, test 2
WS 2017/2018

1) Find all real solutions of the inequality

$$\log_{\frac{1}{3}}(2x^2 + x - 6) \geq \log_{\frac{1}{3}}(4x + 14). \tag{1}$$

Solution: First, we need to find those $x \in \mathbb{R}$ for which logarithms are defined and then solve the inequality. So, we first need to solve

$$2x^2 + x - 6 \quad > \quad 0, \tag{2}$$

$$4x + 14 \quad > \quad 0 \tag{3}$$

Since roots of polynomial in (2) are $-2, \frac{3}{2}$ and leading coefficient is positive, (2) is satisfied for $x \in \mathbb{R} \setminus \left[-2, \frac{3}{2}\right]$. Clearly (3) holds true for $x > -\frac{7}{2}$. So (2) and (3) are satisfied for $x \in M$, where

$$M = \left(-\frac{7}{2}, -2\right) \cup \left(\frac{3}{2}, +\infty\right).$$

Now, we will consider only $x \in M$. Since logarithm with base $\frac{1}{3}$ is decreasing (1) is equivalent to

$$2x^2 + x - 6 \leq 4x + 14.$$

So, we obtain

$$2x^2 - 3x - 20 \leq 0. \tag{4}$$

Since roots of polynomial are $-\frac{5}{2}, 4$ and leading coefficient is positive (4) is satisfied for $x \in \left[-\frac{5}{2}, 4\right]$. Since we consider only $x \in M$ the solution is

$$M \cap \left[-\frac{5}{2}, 4\right] = \left[-\frac{5}{2}, -2\right) \cup \left(\frac{3}{2}, 4\right].$$

$\square$

2) Find all real solutions of the equality

$$3^{x-1} - 2 \cdot 3^{1-x} + 1 = 0. \tag{5}$$

Solution: Put $t = 3^x$. So, (5) can be rewritten as

$$\frac{t}{3} - 2\frac{3}{t} + 1 = 0.$$

Multiplying by $3t$ we obtain

$$t^2 + 3t - 18 = 0.$$

Solutions of this quadratic equality are $-6, 3$. Since $t = 3^x$, t must be positive. Thus $t = 3$ and $x = 1$.

$\square$