Výsledky příkladů
Cvičení 10

1. (a) $f_W(w) = \max(0, 1 - |w|)$
   (b) $f_Z(z) = \max(0, 1 - |z - 1|)$

2. $f_Z(z) = \frac{1}{\theta}\left(\mathrm{e}^{-\lambda \max(0, z-\theta)} - \mathrm{e}^{-\lambda z}\right)$, pro $z > 0$, jinak $f_Z(z) = 0$.
   $\mathsf{E}\, Z = \frac{1}{\lambda} + \frac{\theta}{2}$, $\mathsf{Var}\, Z = \frac{1}{\lambda^2} + \frac{\theta^2}{12}$

3. $f_Z(z) = \frac{1}{\pi}\frac{1}{1+z^2}$, $z \in \mathbb{R}$

4. (a) $Z \sim \Gamma(a, p_1 + p_2)$
   (b) $Z \sim \Gamma(\lambda, n)$
   (c) $Z \sim \chi^2_{\sum_{i=1}^n m_i} \sim \Gamma\left(\frac{1}{2}, \frac{1}{2}\sum_{i=1}^n m_i\right)$

5. $Z \sim \mathsf{N}\left(0, \sigma_1^2 + \sigma_2^2\right)$

6. $f_{\sqrt{X}-\sqrt{Y}}(z) = \frac{2}{5}|z|^5 - 2|z|^3 - 4z^2 + 4|z| + \frac{8}{5}$, pro $-1 < z < 1$, $\mathrm{med}(\sqrt{X} - \sqrt{Y}) = 0$

7. s využitím: $\sin(\theta) = \frac{1}{2\mathrm{i}}\left(\mathrm{e}^{\mathrm{i}\theta} - \mathrm{e}^{-\mathrm{i}\theta}\right)$, $\quad \arctan(\theta) = \frac{\mathrm{i}}{2}\log\left(\frac{1-\mathrm{i}\theta}{1+\mathrm{i}\theta}\right)$

   $U_1 = \mathrm{e}^{-\frac{1}{2}(Z_1^2+Z_2^2)}$, $\quad U_2 = \frac{1}{2\pi}\arctan\left(\frac{Z_1}{Z_2}\right)$

   $f_{Z_1Z_2}(z_1, z_2) = \frac{1}{2\pi}\,\mathrm{e}^{-\frac{1}{2}(z_1^2+z_2^2)} = \frac{1}{\sqrt{2\pi}}\,\mathrm{e}^{-\frac{z_1^2}{2}}\frac{1}{\sqrt{2\pi}}\,\mathrm{e}^{-\frac{z_2^2}{2}} = f_{Z_1}(z_1)\, f_{Z_2}(z_2)$