

UNIVERSAL ALGEBRA I 2021/22: EXAM TOPICS

Note: The list covers *typical* problems that can appear, not *all types* of problems that will appear on exams.

LATTICES

- modularity and distributivity
- complete lattices and closure operators
- algebraic lattices and algebraic closure operators (you can ignore the representation of algebraic lattices by closed sets in algebraic closure operators)
- Galois correspondences

SEMANTICS

- subalgebras, subalgebra generation
- congruences and congruence generation
- quotients, homomorphic images, the 1st and 2nd isomorphism theorems (you can ignore the 3rd one)
- direct product and subpowers, direct decomposition theorem
- H,S,P operators and varieties
- subdirect decomposition, subdirectly irreducible algebras

Typical computational problems (using examples from group theory, unary algebras, binary algebras given by formulas or multiplication tables):

- calculation of Sg, Sub, Cg, Con
- determine all homomorphisms
- identify quotient (1st iso thm)
- direct decomposition
- subdirect decomposition, identify SIs, calculate all SIs in a variety.

SYNTAX

- terms, identities and absolutely free algebras
- free algebras for a class
- the syntax-semantics Galois connection and Birkhoff's theorem
- equational theories as fully invariant congruences

Typical computational problems:

- free algebras in Mod(identities)
- free algebras in HSP(A), A small

CLONES

- functional and relational clones
- relation of clones and free algebras
- the Pol-Inv connection, the Geiger/BKKR Theorem (you can ignore the RelClo=InvPol theorem)
- the Mal'tsev theorem, $CD \cap CP$ (you don't have to remember Jónsson terms and other Mal'tsev conditions)

Typical computational problems:

- calculate $\text{Clo}(A)$, $\text{Pol}(A)$ (generating clones), $f \notin \text{Clo}(A)$, ...
- calculate $\text{Pol}(\mathbb{R})$, $\text{Inv}(\mathbb{F})$