Universal Algebra 1 - Homework 1

Deadline: 26.10.2021, 17:20

1. Let $A = \mathbb{Z}_2 \times \mathbb{Z}_4$ and let $+$ be the standard addition on it. Let K be the subgroup of A generated by $a = (1,0)$ and $b = (0,2)$ and K' its complement. We define two multiplications on A by

$$x *_1 y = \begin{cases} a \text{ if } x, y \in K' \\ (0,0) \text{ else.} \end{cases} \qquad x *_2 y = \begin{cases} b \text{ if } x, y \in K' \\ (0,0) \text{ else.} \end{cases}$$

   Both $(A, +, *_1, 0, -)$ and $(A, +, *_2, 0, -)$ are rings. Show that their *reducts* $(A, *_1)$ and $(A, *_2)$ are isomorphic, but they are not isomorphic as rings.

2. Let $\mathbf{Eq}(X)$ be the lattice of equivalence relations on a set X.

   - Show that for all $\alpha, \beta \in \mathbf{Eq}(X)$: $\alpha \circ \beta \subseteq \alpha \vee \beta$, but that equality does not hold in general.
   - Show that $\alpha \vee \beta = \alpha \cup (\alpha \circ \beta) \cup (\alpha \circ \beta \circ \alpha) \cup \ldots$