Orthogonality and complementation in the lattice of subspaces of a finite vector space

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Abstract

We investigate the lattice $L(V)$ of subspaces of an m-dimensional vector space V over a finite field $GF(q)$ with a prime power $q = p^n$ together with the unary operation of orthogonality. It is well-known that this lattice is modular and that the orthogonality is an antitone involution. The lattice $L(V)$ satisfies the chain condition and we determine the number of covers of its elements, especially the number of its atoms. We characterize when orthogonality is a complementation and hence when $L(V)$ is orthomodular. For q being a prime number and $m = 2$ we characterize orthomodularity of $L(V)$ by a simple condition.