Selected topics in harmonic analysis 2 – SS 2023 - 2024
Information about the exam

The exam will be oral and can be held either in English or in Czech, depending on the preference of the student. Please contact me in order to schedule a time for your exam.

The exam will consist of three questions:

(i) one definition; (ii) one theorem from the theory of weighted inequalities; (iii) one theorem from the theory of vector-valued singular integrals/ Littlewood-Paley theory/ Fourier multiplier theorems.

A list of possible topics for each of these questions is listed below. After stating the theorems in parts (ii) and (iii), you will be asked about the proof of one of the theorems. Proofs will be required only to the extent in which they were presented in the lectures and emphasis will be put on the understanding of the basic idea of the proof, rather than on technical details.

(i)

A_1 weight
A_p weight $(1 < p < \infty)$
A_∞ weight
reverse Hölder class
Littlewood-Paley operator and Littlewood-Paley square function
Riesz potential
Bessel potential

(ii)

Weighted weak-type estimate for the Hardy-Littlewood maximal operator (Proposition 9)
Weighted strong-type estimate for the Hardy-Littlewood maximal operator (Theorem 12)
Reverse Hölder inequality (Theorem 13)
Self-improving properties of A_p weights (Corollary 15)
Characterization of A_1 weights (Proposition 23)
Factorization of A_p weights (Proposition 24 and Theorem 25)
Rubio de Francia iteration algorithm (Theorem 26)
Rubio de Francia extrapolation of boundedness (Theorem 27)

(iii)

Vector-valued Calderón-Zygmund theorem (Theorem 28)
Littlewood-Paley theorem (Theorem 32)
Lower Littlewood-Paley inequality (Theorem 36)
Mikhlin multiplier theorem (Theorem 41)
Marcinkiewicz multiplier theorem (Theorem 43; without proof)