① $(2-e^x)\, y' = -3e^x \cdot \operatorname{tg} y \cdot \cos^2 y$

$$\frac{y'}{\operatorname{tg} y \cdot \cos^2 y} = \frac{-3e^x}{2-e^x} \quad \Big/ \int (\dots)\, dx$$

$[x \neq \ln 2,\ y \neq k\frac{\pi}{2},\ k \in \mathbb{Z}]$

L.S.: $\displaystyle\int \frac{dy}{\operatorname{tg} y \cdot \cos^2 y} = \int \frac{\frac{1}{\cos^2 y}\, dy}{\operatorname{tg} y} \overset{c}{=} \ln|\operatorname{tg} y|$

P.S.: $\displaystyle\int \frac{-3e^x\, dx}{2-e^x} = \int \frac{-3\, dt}{2-t} \overset{c}{=} 3 \ln|2-t|$

$[\text{SUBST}: \ t = e^x,\ dt = e^x\, dx]$

$$\ln|\operatorname{tg} y| = 3\ln|2-e^x| + C \quad , \ C \in \mathbb{R}$$

$$|\operatorname{tg} y| = \exp\left(\ln|2-e^x|^3 + C\right)$$

$$|\operatorname{tg} y| = e^C \cdot |2-e^x|^3 \qquad K := \pm e^C$$

$$\operatorname{tg} y = K \cdot (2-e^x)^3 \quad , \ K \in \mathbb{R}$$

na intervalech, kde $2-e^x \neq 0$, tj. $[x \neq \ln 2]$, tj. $x \in (-\infty, \ln 2)$ nebo $x \in (\ln 2, \infty)$.

Na těchto intervalech je $\operatorname{sgn}(2-e^x)$ konstantní, takže mohu odstranit absolutní h. i vpravo.

$$\operatorname{tg} y = K \cdot (2-e^x)^3$$

$$\operatorname{arctg}(\operatorname{tg} y) = \operatorname{arctg}\left(K \cdot (2-e^x)^3\right)$$

$$y(x) - k\pi = \operatorname{arctg}\left(K \cdot (2-e^x)^3\right), \quad k \in \mathbb{Z}$$

$$y(x) = \operatorname{arctg}\left(K \cdot (2-e^x)^3\right) + k\pi$$

můžeme slepit v bodě $x = \ln 2$.

Řešení je def. na celém $\mathbb{R}$.

Pro $K = 0$ a různé volby $k \in \mathbb{Z}$ dostáváme všechna stacionární řešení:

$$y \equiv k\pi \quad \text{na } \mathbb{R}, \quad k \in \mathbb{Z}.$$

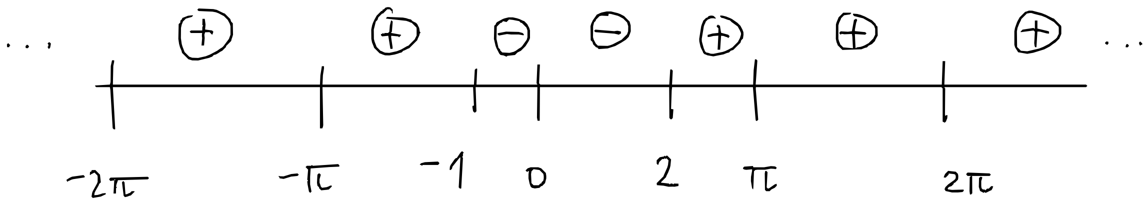
Pozn.: $y = \frac{\pi}{2} + k\pi$ jsou sice nulové body funkce $\cos y$, ale nedají se dosadit do $\operatorname{tg} y$, a proto to nejsou S.Ř. naší rovnice.

(2.) $y' = (y^2 - y - 2)\cdot\sqrt{|\sin y|} =: g(y)$

STACIONÁRNÍ ŘEŠENÍ: $y \equiv k\pi$, $k \in \mathbb{Z}$

$y^2 - y - 2 = (y-2)(y+1)$ … $y \equiv 2$, $y \equiv -1$

MONOTONIE: Zn. derivace závisí pouze na $(y^2 - y - 2)$, protože $\sqrt{|\sin y|} \geq 0$, $y \in \mathbb{R}$.



Lepení na 2+: $\int_2^3 \frac{1}{g}$ K. ?

Srovnáme s $\int_2^3 h$, kde $h(y) = \frac{1}{y-2}$:

$$\lim_{y \to 2+} \frac{\frac{1}{g(y)}}{h(y)} = \lim_{y \to 2+} \left((y+1)\sqrt{|\sin y|} \right)^{-1} =$$

$$= \frac{1}{3\sqrt{\sin 2}} \in (0, \infty).$$

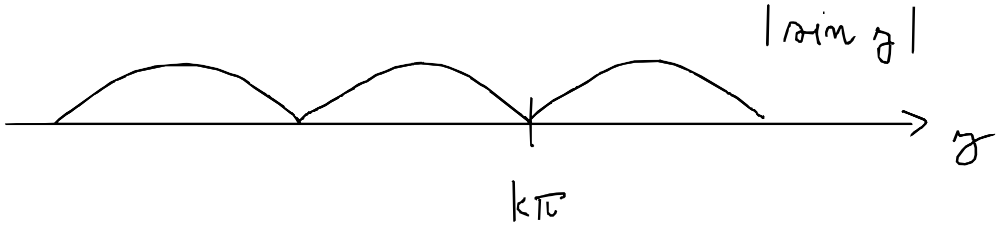
LSK $\Rightarrow \left[\int_2^3 \frac{1}{g} \text{ K.} \Leftrightarrow \int_2^3 h \text{ K.} \right]$ ale $\int_2^3 \frac{dy}{y-2} = \int_0^1 \frac{dy}{y} = \infty$.

Tedy nelze lepit na 2+.

Analogicky: 2−.

Dále (analogicky), srovnáním s $\frac{1}{y+1}$ zjistíme, že nelze lepit ani na $1\pm$.

Lepení na $k\pi_+$: $\displaystyle\int_{k\pi}^{k\pi+1} \frac{1}{g}$ K. ?



Lokálně u $k\pi$ se $|\sin y|$ chová jako $|y-k\pi|$. Tedy $\dfrac{1}{\underbrace{\cdots}\sqrt{|\sin y|}}$ srovnáme

(u $k\pi$ nehraje roli)

s funkcí $\dfrac{1}{\sqrt{|y-k\pi|}} =: h(y)$.

$$\lim_{y\to k\pi_+} \frac{\frac{1}{g(y)}}{h(y)} = \lim_{y\to k\pi_+} \frac{\sqrt{|y-k\pi|}}{(y^2-y-2)\sqrt{|\sin y|}}$$

$$\overset{\text{VOAL}}{\underset{\text{VOLSF}}{=}} \frac{1}{((k\pi)^2-k\pi-2)} \cdot \sqrt{\left|\underbrace{\lim_{y\to k\pi_+} \frac{y-k\pi}{\sin y}}\right|} \in \mathbb{R}\setminus\{0\}.$$

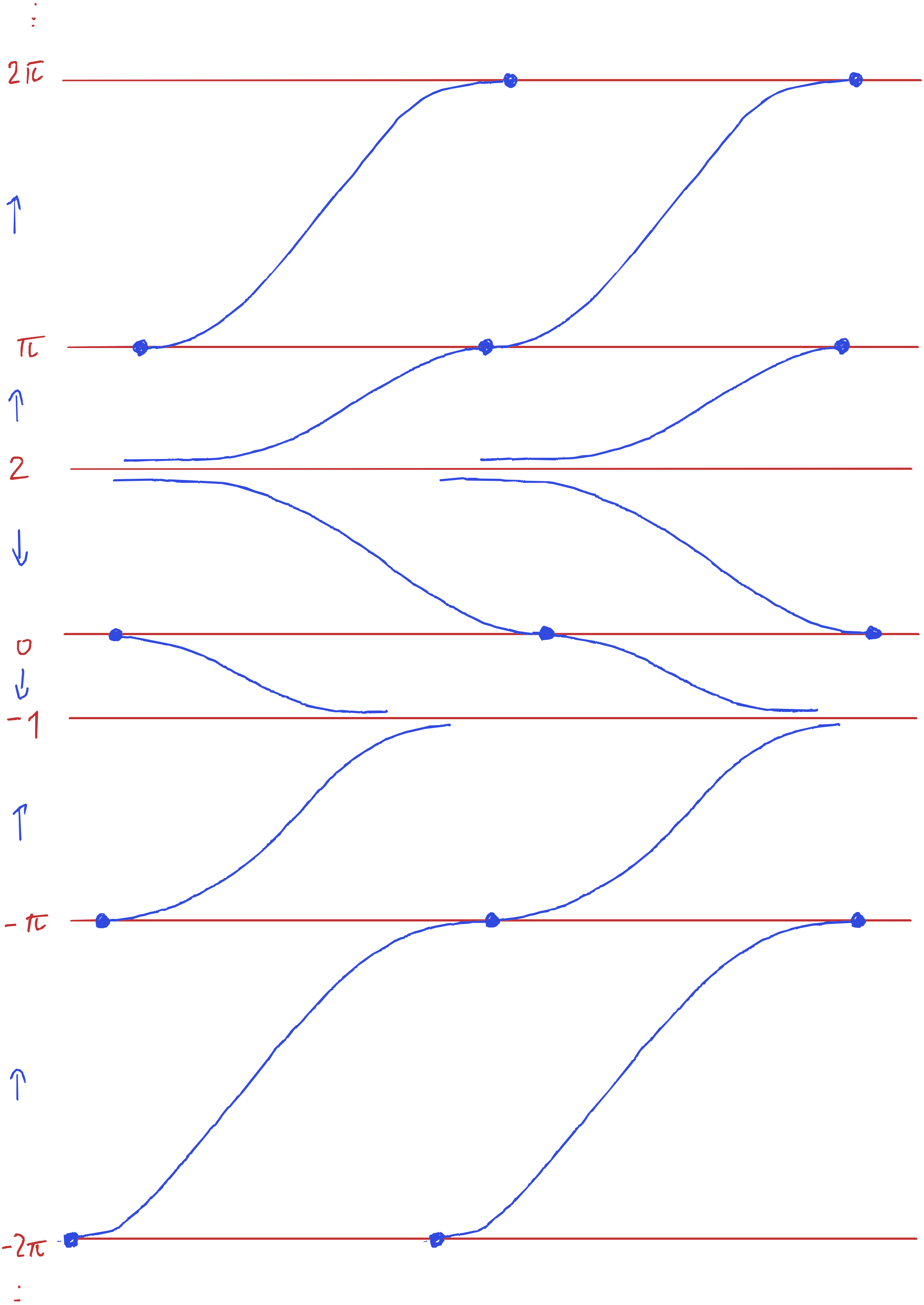
$$\left[\overset{\text{VOAL}}{\underset{\text{VOLSF}}{=}} \left(\lim_{t\to 0_+} \frac{\sin(t+k\pi)}{t}\right)^{-1} = \left(\lim_{t\to 0_+} \frac{\pm\sin t}{t}\right)^{-1} = \pm 1\right.$$

$$\text{LSK} \Rightarrow \left[\int_{k\pi}^{k\pi+1} \frac{1}{g}\ \text{K.} \iff \int_{k\pi}^{k\pi+1} h\ \text{K.}\right].$$

Ovšem $$\int_{k\pi}^{k\pi+1} h = \int_{k\pi}^{k\pi+1} \frac{1}{\sqrt{|y-k\pi|}}\,dy = \int_0^1 \frac{dt}{\sqrt{t}} =$$

$$= \left[2\sqrt{t}\right]_0^1 = 2.$$ Tedy $\displaystyle\int_{k\pi}^{k\pi+1} \frac{1}{g}$ K.,

a lze lepit na $k\pi_+$ (analog. $k\pi_-$).



2π
π
2
0
-1
-π
-2π

(3.) $$y' + \frac{x}{1+x^2} \cdot y = \sqrt{x^2+1} \cdot \cos x$$

$$p(x) = \frac{x}{1+x^2}$$

$$P(x) \stackrel{c}{=} \int \frac{x\,dx}{1+x^2} = \frac{1}{2}\int \frac{2x\,dx}{1+x^2} = \overbrace{\frac{1}{2}\ln|1+x^2|}^{=\ln\sqrt{1+x^2}}$$

$$\text{I.F.}: \quad e^{\ln\sqrt{1+x^2}} = \sqrt{1+x^2}$$

$$y'\sqrt{1+x^2} + \frac{x}{\sqrt{1+x^2}}\, y = (x^2+1)\cos x$$

$$\left(y\sqrt{1+x^2}\right)' = (x^2+1)\cos x \quad \Big/ \int(\dots)\,dx$$

P.S.: $$\int \overset{\downarrow}{x^2}\,\overset{\uparrow}{\cos x} \stackrel{\text{P.P.}}{=} x^2\sin x - \int \overset{\downarrow}{2x}\,\overset{\uparrow}{\sin x}\,dx =$$

$$= x^2\sin x - \left(-2x\cos x - \int 2(-\cos x)\,dx\right)$$

$$= x^2\sin x + 2x\cos x - 2\int \cos x\,dx$$

$$\stackrel{c}{=} x^2\sin x + 2x\cos x - 2\sin x$$

$$y\sqrt{1+x^2} = x^2\sin x + 2x\cos x - 2\sin x + \sin x + C$$

$$= (x^2-1)\cdot\sin x + 2x\cos x + C$$

$$y(x) = \frac{1}{\sqrt{x^2+1}}\left((x^2-1)\sin x + 2x\cos x + C\right),$$

$x \in \mathbb{R}$ $\quad (C \in \mathbb{R})$.

3. $y''' = y \qquad y''' - y = 0$

$$\lambda^3 - 1 = (\lambda - 1)(\lambda^2 + \lambda + 1)$$

$$\frac{-1 \pm \sqrt{1^2 - 4}}{2} = -\frac{1}{2} \pm i \cdot \frac{\sqrt{3}}{2}$$

$$F.S. := \left\{ e^{t},\ e^{-\frac{t}{2}} \cdot \cos\left(\frac{\sqrt{3}}{2} t\right),\ e^{-\frac{t}{2}} \sin\left(\frac{\sqrt{3}}{2} t\right) \right\}.$$

Obecné řešení rovnice je tedy tvaru:

$$y(x) = Ae^{-t} + B \cdot e^{-\frac{t}{2}} \cos\left(\frac{\sqrt{3}}{2} t\right) + Ce^{-\frac{t}{2}} \sin\left(\frac{\sqrt{3}}{2} t\right)$$

$(A, B, C \in \mathbb{R})$, $t \in \mathbb{R}$.