1. $$\lim_{x\to 0} \frac{\ln(1+x^2) - x^2\cdot\cos x}{x^2 - \sin(x^2)} =$$

$$\sin y = y - \frac{y^3}{6} + o(y^4),\quad y\to 0$$

$$\sin x^2 = x^2 - \frac{x^6}{6} + o(x^8),\quad x\to 0$$

jmenovatel: $\frac{x^6}{6} + o(x^6),\quad x\to 0$

$$\ln(1+y) = y - \frac{y^2}{2} + \frac{y^3}{3} + o(y^3),\quad y\to 0$$

$$\ln(1+x^2) = x^2 - \frac{x^4}{2} + \frac{x^6}{3} + o(x^6),\quad x\to 0$$

$$x^2\cdot\cos x = x^2\cdot\left(1 - \frac{x^2}{2} + \frac{x^4}{24} + o(x^5)\right),\quad x\to 0$$

$$= x^2 - \frac{x^4}{2} + \frac{x^6}{24} + o(x^6),\quad x\to 0.$$

$$= \lim_{x\to 0} \frac{x^2 - \frac{x^4}{2} + \frac{x^6}{3} - \left(x^2 - \frac{x^4}{2} + \frac{x^6}{24}\right) + o(x^6)}{\frac{x^6}{6} + o(x^6)} =$$

$$= \lim_{x\to 0} \frac{x^6\left(\frac{1}{3} - \frac{1}{24} + \frac{o(x^6)}{x^6}\right)}{x^6\left(\frac{1}{6} + \frac{o(x^6)}{x^6}\right)} = \frac{\frac{7}{24} + 0}{\frac{1}{6} + 0} = \frac{7}{4}$$

(2.) $\displaystyle\int \frac{3\sin x\cos x + \cos x}{\cos^2 x - \sin x\cos^2 x - 3}\,dx =$

Lichá v cosinu $\Rightarrow \left[y = \sin x\ ,\ dy = \cos x\,dx \right]$

3

$$\int \frac{(3\sin x + 1)\cos x\,dx}{1-\sin^2 x - \sin x\,(1-\sin^2 x) - 3} = \int \frac{(3y+1)\,dy}{y^3 - y^2 - y - 2}$$

$y^3 - y^2 - y - 2 = 0$ ... zkusmo : $y = 2$ kořen

2

$$(y^3 - y^2 - y - 2) : (y-2) = y^2 + y + 1$$
$$\frac{-(y^3 - 2y^2)}{\quad}$$
$$y^2 - y - 2$$
$$\frac{-(y^2 - 2y)}{\quad}$$
$$y - 2$$

Tedy $y^3 - y^2 - y - 2 = (y^2 + y + 1)(y - 2)$

PARCIÁLNÍ ZLOMKY : $\dfrac{3y+1}{(y^2+y+1)(y-2)} =$

5

$$= \frac{Ay + B}{y^2 + y + 1} + \frac{C}{y-2} = \frac{-y}{y^2+y+1} + \frac{1}{y-2}$$

$$= \frac{1}{\dots}\left(Ay^2 - 2Ay + By - 2B + C(y^2 + y + 1)\right)$$

y^2 : $A + C = 0$

y : $-2A + B + C = 3$

1 : $-2B + C = 1$

$$\left(\begin{array}{ccc|c} 1 & 0 & 1 & 0 \\ -2 & 1 & 1 & 3 \\ 0 & -2 & 1 & 1 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 0 & 1 & 0 \\ 0 & 1 & 3 & 3 \\ 0 & -2 & 1 & 1 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 0 & 1 & 0 \\ 0 & 1 & 3 & 3 \\ 0 & 0 & 7 & 7 \end{array}\right)$$

$$\boxed{C = 1\ ,\quad B = 0\ ,\quad A = -1}$$

$$I = \int \frac{(3y+1)\,dy}{y^3-y^2-y-2} = \int\left(\frac{-y}{y^2+y+1}+\frac{1}{y-2}\right)dy$$

$$= \underbrace{-\frac{1}{2}\int\frac{2y+1-1}{y^2+y+1}\,dy}_{6} + \underbrace{\ln|y-2|}_{2} =$$

$$= -\frac{1}{2}\int\frac{2y+1}{y^2+y+1}\,dy + \frac{1}{2}\int\frac{dy}{\left(y+\frac{1}{2}\right)^2+\frac{3}{4}} + \ln|y-2|$$

$$= -\frac{1}{2}\ln|y^2+y+1| + \ln|y-2| + \frac{1}{2}\cdot\frac{4}{3}\int\frac{dy}{\left(\frac{2y+1}{\sqrt{3}}\right)^2+1}$$

1

$$\overset{c}{=} -\frac{1}{2}\ln\left(y^2+y+1\right) + \ln|y-2| + \frac{1}{2}\cdot\frac{4}{3}\cdot\frac{\sqrt{3}}{2}\operatorname{arctg}\left(\frac{2y+1}{\sqrt{3}}\right)$$

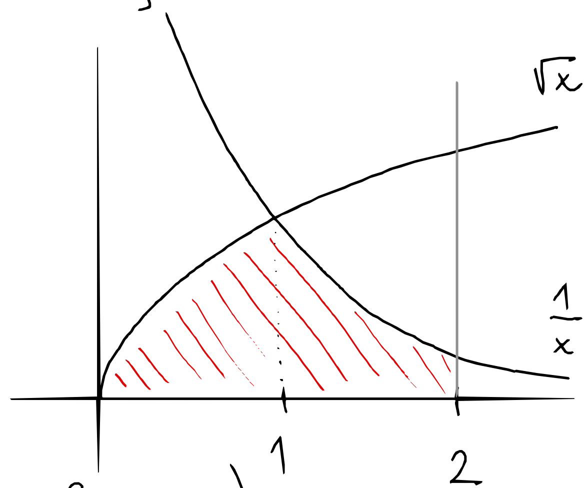
1

$$= -\frac{1}{2}\ln\left(\sin^2 x+\sin x+1\right) + \ln|\sin x-2| +$$

$$+ \frac{\sqrt{3}}{3}\operatorname{arctg}\left(\frac{2\sin x+1}{\sqrt{3}}\right)$$

③ $f(x) = \min\left\{\sqrt{x}, \frac{1}{x}\right\}, \quad x \in [0,2]$

$V = \pi \int_0^2 f^2$



$$V = \pi\left(\int_0^1 (\sqrt{x})^2 dx + \int_1^2 \left(\frac{1}{x}\right)^2 dx\right) =$$

$$= \pi\left(\left[\frac{x^2}{2}\right]_0^1 + \left[\frac{-1}{x}\right]_1^2\right) =$$

$$= \pi\left(\left(\frac{1^2}{2} - 0\right) + \left(-\frac{1}{2} - (-1)\right)\right) =$$

$$= \pi\left(\frac{1}{2} + \frac{1}{2}\right) = \underline{\underline{\pi}}$$