Remeshing Strategy in ALE method: Contactless Rebound Simulation

ERC-CZ Grant LL2105 CONTACT

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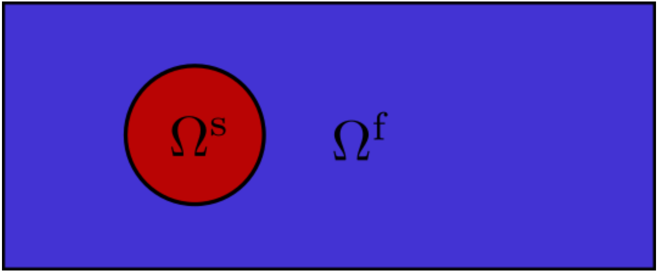
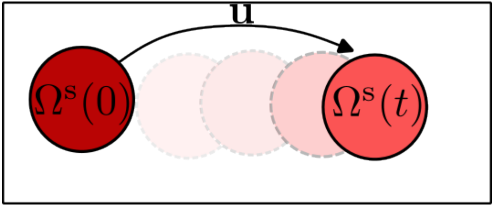
Problem Description



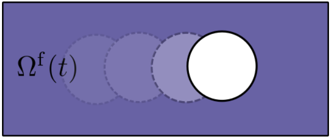
Solid: Lagrangian description



- Lagrangian formulation
- 1st Piola–Kirchhoff stress tensor $\mathbb{P}$

$$\rho^{\mathrm{s}} \int_{\Omega^{\mathrm{s}}(0)} \partial_{tt}\mathbf{u} \cdot \varphi_{\mathrm{u}} \,\mathrm{d}\mathbf{x} = \int_{\Omega^{\mathrm{s}}(0)} \mathbb{P}(\mathbf{u}) \cdot \nabla \varphi_{\mathrm{u}} \,\mathrm{d}\mathbf{x} \tag{1}$$

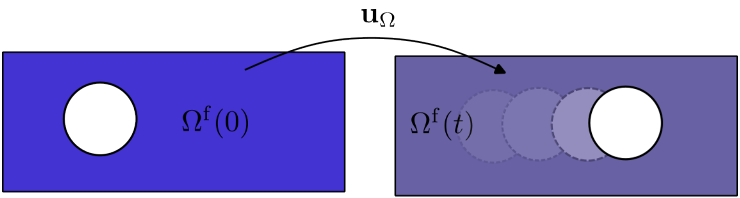
Arbitrary Lagrangian-Eulerian method



- Moving domain $\Omega(t)$
- Incompressible Navier-Stokes material

$$\rho^{\mathrm{f}} \int_{\Omega^{\mathrm{f}}(t)} \partial_t \mathbf{v} \cdot \varphi_{\mathrm{v}} \, \mathrm{d}\mathbf{x} + \rho^{\mathrm{f}} \int_{\Omega^{\mathrm{f}}(t)} \nabla \mathbf{v} \mathbf{v} \cdot \varphi_{\mathrm{v}} \, \mathrm{d}\mathbf{x} = \int_{\Omega^{\mathrm{f}}(t)} \mathbb{T}(\mathbf{v}, p) \cdot \nabla \varphi_{\mathrm{v}} \, \mathrm{d}\mathbf{x}$$
$$\int_{\Omega^{\mathrm{f}}(t)} \mathrm{div}(\mathbf{v}) \varphi_{\mathrm{p}} = 0$$
$$\mathbb{T} = \mu \left(\nabla \mathbf{v} + (\nabla \mathbf{u})^T \right) - p\mathbb{I} \tag{2}$$

Arbitrary Lagrangian-Eulerian method



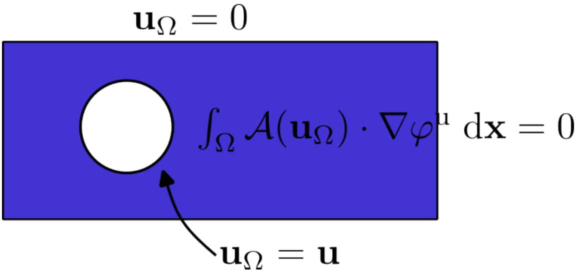
- $\mathbf{u}_\Omega$ denotes the displacement of domain $\Omega^f(0)$ to $\Omega^f(t)$
- $\hat{\mathbb{F}} := \mathbb{I} + \hat{\nabla}\mathbf{u}_\Omega$
- $\phi_\Omega(\hat{\mathbf{x}}, t) := \hat{\mathbf{x}} + \mathbf{u}_\Omega(\hat{\mathbf{x}}, t)$
- $\hat{\mathbf{v}}(\hat{\mathbf{x}}, t) := \mathbf{v}(\phi_\Omega(\hat{\mathbf{x}}, t), \hat{\mathbf{x}})$
- $\hat{p}(\hat{\mathbf{x}}, t) := p(\phi_\Omega(\hat{\mathbf{x}}, t), \hat{\mathbf{x}})$
- $\int_{\Omega^f(t)} f(\mathbf{x}, t)\,\mathrm{d}\mathbf{x} = \int_{\Omega^f(0)} \det(\hat{\mathbb{F}})\hat{f}(\hat{\mathbf{x}}, t)\,\mathrm{d}\hat{\mathbf{x}}$

Arbitrary Lagrangian-Eulerian method

$$
\begin{aligned}
&\rho^{\mathrm{f}} \int_{\Omega^{\mathrm{f}}(0)} \det(\hat{\mathbb{F}}) \partial_t \hat{\mathbf{v}} \cdot \varphi_{\mathrm{v}} \, \mathrm{d}\hat{\mathbf{x}} \\
&+ \rho^{\mathrm{f}} \int_{\Omega^{\mathrm{f}}(0)} \det(\hat{\mathbb{F}}) (\hat{\nabla} \hat{\mathbf{v}} \hat{\mathbb{F}}^{-1})(\hat{\mathbf{v}} - \partial_t \mathbf{u}_{\Omega}) \cdot \varphi_{\mathrm{v}} \, \mathrm{d}\hat{\mathbf{x}} \\
&= \int_{\Omega^{\mathrm{f}}(0)} \det(\hat{\mathbb{F}}) \mathbb{T}(\hat{\mathbf{v}}, \hat{p}) \cdot \hat{\nabla} \varphi_{\mathrm{v}} \hat{\mathbb{F}}^{-1} \, \mathrm{d}\hat{\mathbf{x}} \\
&\int_{\Omega^{\mathrm{f}}(0)} \det(\hat{\mathbb{F}}) \mathrm{tr}(\hat{\nabla} \hat{\mathbf{v}} \hat{\mathbb{F}}^{-1}) \varphi_{\mathrm{p}} \, \mathrm{d}\hat{\mathbf{x}} = 0
\end{aligned}
\tag{3}
$$

- $\mathbb{T} = 2\mu(\hat{\nabla} \hat{\mathbf{v}} \hat{\mathbb{F}}^{-1} + \hat{\mathbb{F}}^{-T} (\hat{\nabla} \hat{\mathbf{v}})^T) - \hat{p}\mathbb{I}$

Arbitrary Lagrangian-Eulerian method



- We are looking for $\mathbf{u}_\Omega$
- $\mathbf{u}_\Omega = 0$ at $\partial\Omega$
- $\mathbf{u}_\Omega = \mathbf{u}$ at Γ

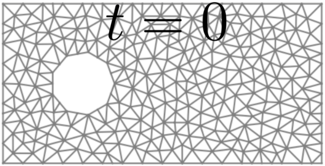
$$\int_{\Omega^{\mathrm{f}}(0)} \mathcal{A}(\mathbf{u}_\Omega) \cdot \hat{\nabla}\varphi_{\mathrm{u}} \, \mathrm{d}\hat{\mathbf{x}} = 0 \tag{4}$$

Arbitrary Lagrangian-Eulerian method

$$
\begin{aligned}
&\rho^{\mathrm{f}} \int_{\Omega^{\mathrm{f}}(0)} \det(\mathbb{F}) \partial_t \hat{\mathbf{v}} \cdot \varphi_{\mathrm{v}} \, \mathrm{d}\hat{\mathbf{x}} \\
&+ \rho^{\mathrm{f}} \int_{\Omega^{\mathrm{f}}(0)} \det(\mathbb{F}) (\hat{\nabla} \hat{\mathbf{v}} \mathbb{F}^{-1})(\hat{\mathbf{v}} - \partial_t \mathbf{u}_{\Omega}) \cdot \varphi_{\mathrm{v}} \, \mathrm{d}\hat{\mathbf{x}} \\
&= \int_{\Omega^{\mathrm{f}}(0)} \det(\mathbb{F}) \mathbb{T}(\hat{\mathbf{v}}, \hat{p}) \cdot \hat{\nabla} \varphi_{\mathrm{v}} \mathbb{F}^{-1} \, \mathrm{d}\hat{\mathbf{x}} \\
&\int_{\Omega^{\mathrm{f}}(0)} \det(\mathbb{F}) \mathsf{tr}(\hat{\nabla} \hat{\mathbf{v}} \mathbb{F}^{-1}) \varphi_{\mathrm{p}} \, \mathrm{d}\hat{\mathbf{x}} = 0 \\
&\int_{\Omega^{\mathrm{f}}(0)} \mathcal{A}(\mathbf{u}_{\Omega}) \cdot \varphi_{\mathrm{u}} \, \mathrm{d}\hat{\mathbf{x}} = 0 \\
&\rho^{\mathrm{s}} \int_{\Omega^{\mathrm{s}}(0)} \partial_{tt} \mathbf{u} \cdot \varphi_{\mathrm{u}} \, \mathrm{d}\mathbf{x} = \int_{\Omega^{\mathrm{s}}(0)} \mathbb{P}(\mathbf{u}) \cdot \nabla \varphi_{\mathrm{u}} \, \mathrm{d}\mathbf{x} \\
&\mathbf{u} = \mathbf{u}_{\Omega} \text{ at } \Gamma \\
&\partial_t \mathbf{u} = \hat{\mathbf{v}} \text{ at } \Gamma \\
&\mathbb{P}\mathbf{n} = \det(\hat{\mathbb{F}}) \mathbb{T} \hat{\mathbb{F}}^{-T} \mathbf{n} \text{ at } \Gamma
\end{aligned}
\tag{5}
$$

Arbitrary Lagrangian-Eulerian method: Re-meshing

- The displacement $\mathbf{u}_\Omega$ can violate the mesh regularity
- It is possible to increase the regularity by re-meshing the process
- Additional interpolation is necessary



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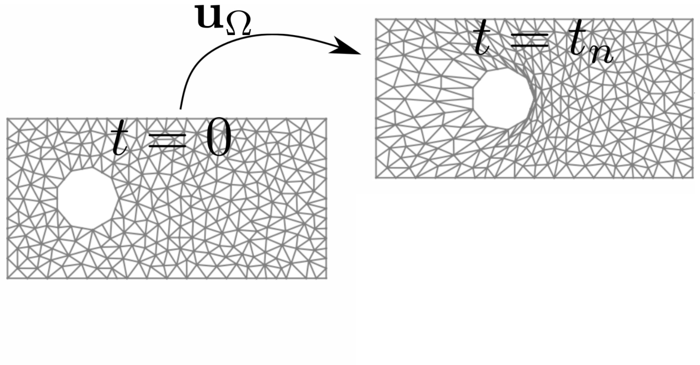

Figure 1: Update with re-meshing.

Arbitrary Lagrangian-Eulerian method: Re-meshing

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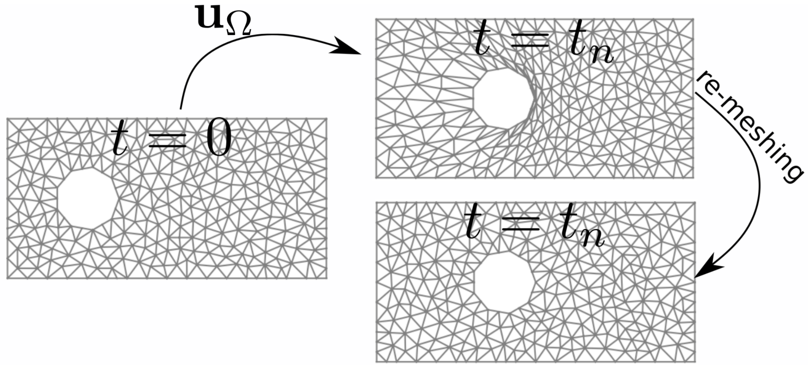
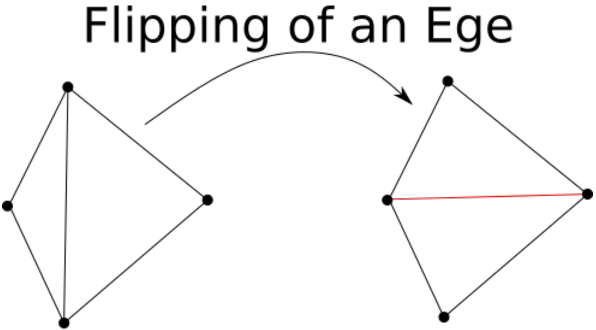


Figure 1: Update with re-meshing.

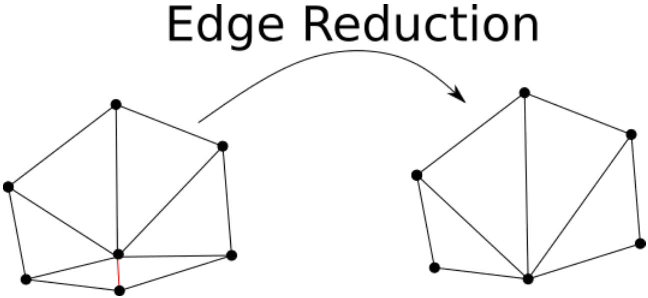
- We will change the mesh locally where it is needed
- We need to keep the interface
- The mesh can be built just ones
- The number of operations is $\mathcal{O}(n)$, where n is number of elements

ADmesh

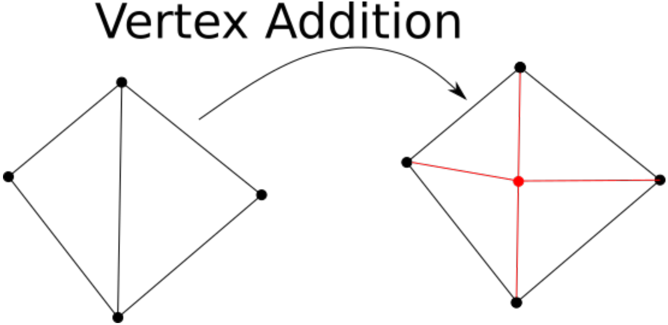
Arbitrary Lagrangian-Eulerian method: Local mesh operations



- Flipping of an Edge

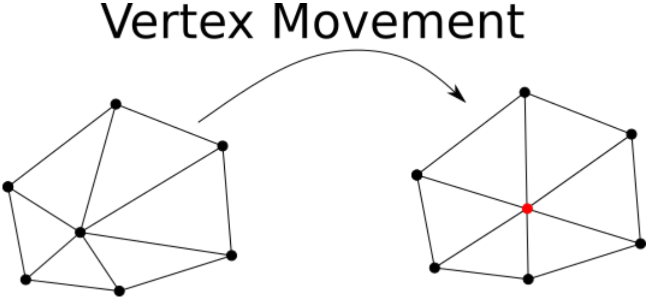


- Flipping of an Edge
- Edge reduction



Arbitrary Lagrangian-Eulerian method: Local mesh operations

- Flipping of an Edge
- Edge reduction
- Vertex Addition



Rebound: Definition

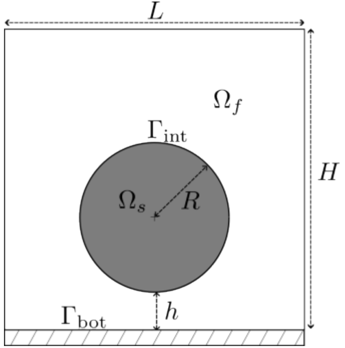


Figure 2: Geometry of the problem

R	ball radius	0.2 m
h	initial distance	0.1 m
H	domain height	0.8 m
L	domain length	0.8 m

[1] J. Fara, S. Schwarzacher, and K. Tůma. **"Geometric re-meshing strategies to simulate contactless rebounds of elastic solids in fluids".** In: *Computer Methods in Applied Mechanics and Engineering* 422 (2024), p. 116824.

Rebound: Equations

- Navier-Stokes for fluid
- Neo-Hokean for solid
- initial velocity $\mathbf{v}_{\mathrm{s}}(0, x, y) = (0, -0.5)$; $\mathbf{v}_{\mathrm{f}}$ solution to steady Stokes poroblem.
- $\mathbf{v}(t, x, 0) = 0$ on $\mathit{\Gamma}_{\mathrm{bot}}$

label	description	units	value
ρ_f	fluid density	$kg\, m^{-3}$	1
ρ_s	solid density	$kg\, m^{-3}$	1000
μ	fluid viscosity	$Pa\, s$	0.1, 0.01, 0.001
G	elastic modulus	kPa	50

Rebound: Measured quantities

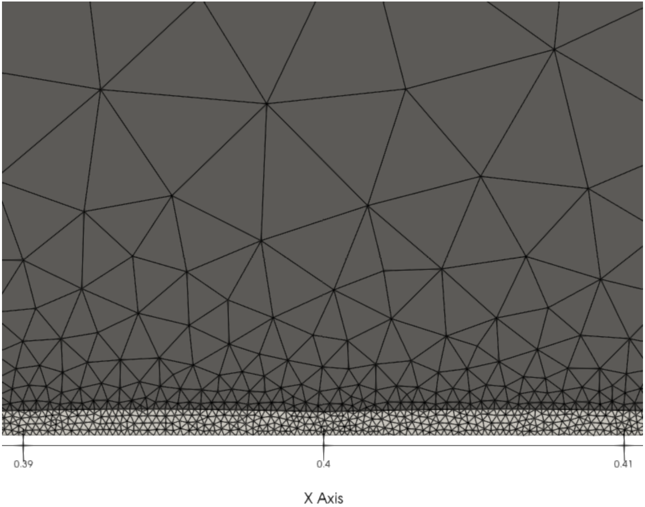
$$y_{\mathrm{min,c}} = \min_{(0.4,y)\in\Omega_{\mathrm{s}}} y, \qquad y_{\mathrm{min}} = \min_{(x,y)\in\Omega_{\mathrm{s}}} y, \tag{6}$$

$$p_{\mathrm{bc}} = p([0.4, 0.0], t), \qquad E_{\mathrm{k,s}} = \int_{\Omega_{\mathrm{s}}} \frac{\rho_{\mathrm{s}}}{2} |\mathbf{v}_0|^2 \, \mathrm{d}\mathbf{x}, \tag{7}$$

$$E_{\mathrm{el,s}} = \int_{\Omega_{\mathrm{s}}} \frac{G}{2} \left(\mathsf{tr}(\mathbb{F}\mathbb{F}^T) - 2 \right) \, \mathrm{d}\mathbf{x}, \qquad E_{\mathrm{s}} = E_{\mathrm{k,s}} + E_{\mathrm{el,s}}. \tag{8}$$

Rebound: Refining strategy

- Eiconal refinement



mesh	mesh_0^{200}	mesh_1^{200}	mesh_2^{200}	mesh_3^{200}	mesh_4^{200}
$\#cells$	7956	15165	25046	37550	52153

Table 1: Number of cells in the meshes.

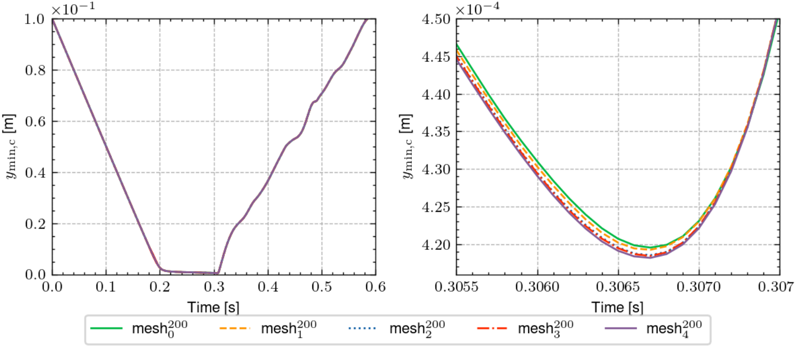
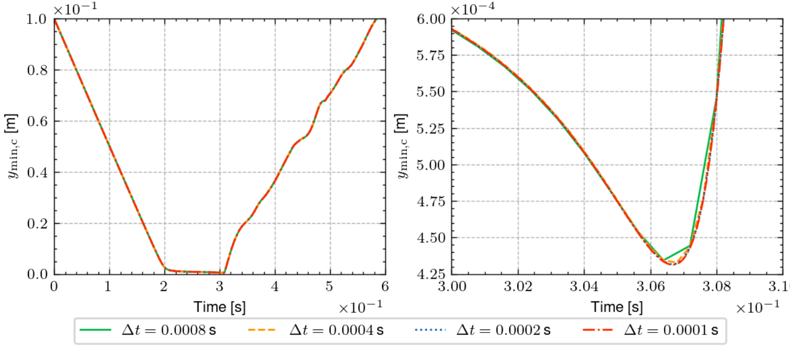


Figure 3: Time convergence. Mesh mesh_4^{200}

Rebound: Time convergence



$\Delta t[s]$	8×10^{-4}	4×10^{-4}	2×10^{-4}	1×10^{-4}
$\min_t y_{\mathrm{min,c}}$ [m]	4.361×10^{-4}	4.338×10^{-4}	4.332×10^{-4}	4.330×10^{-4}
$\max_t p_{\mathrm{bc}}$ [Pa]	23068.022	23070.493	23068.551	23065.842
$\max_t E_{\mathrm{el,s}}$ [J]	11.220	11.220	11.220	11.220
$\min_t E_{\mathrm{k,s}}$ [J]	8.966×10^{-2}	8.760×10^{-2}	8.757×10^{-2}	8.755×10^{-2}

Rebound: Energy

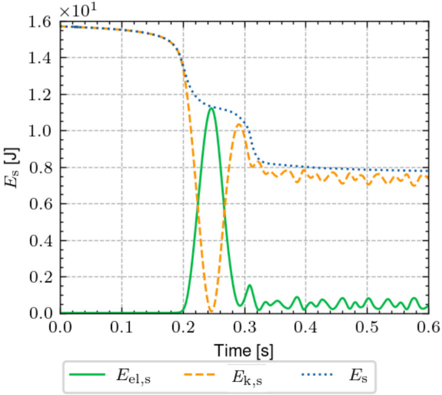


Figure 4: Energy

Rebound: Pressure

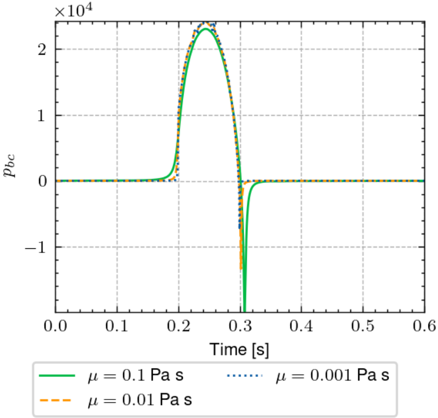


Figure 5: Pressure

Rebound: μ convergence

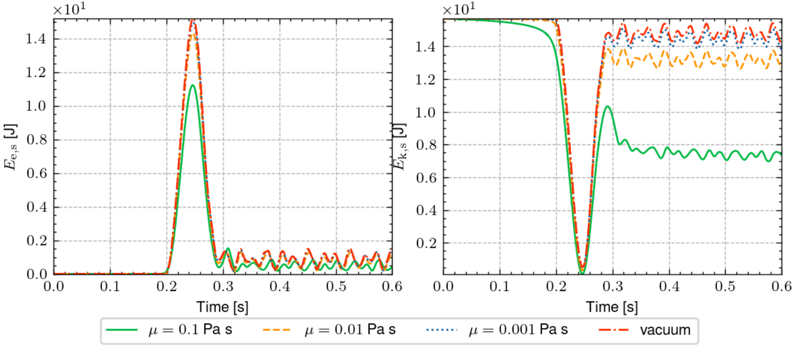


Figure 6: Energies

Rebound: μ convergence

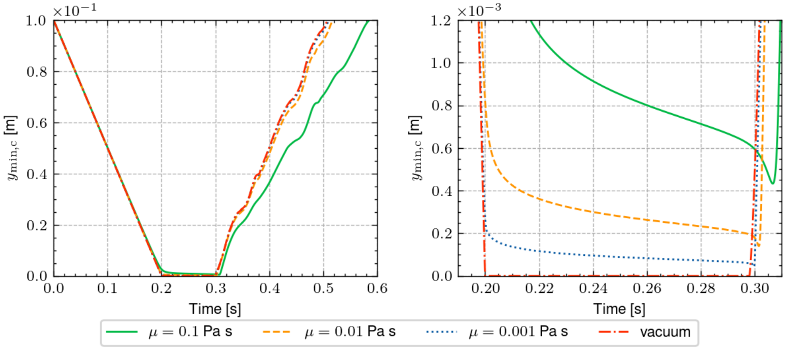


Figure 7: Minimum

Rebound: μ convergence

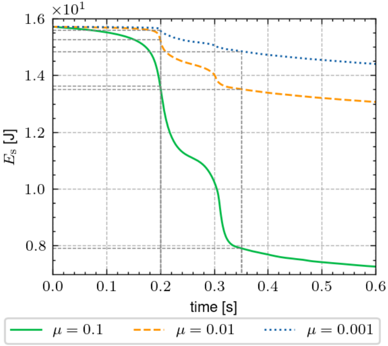
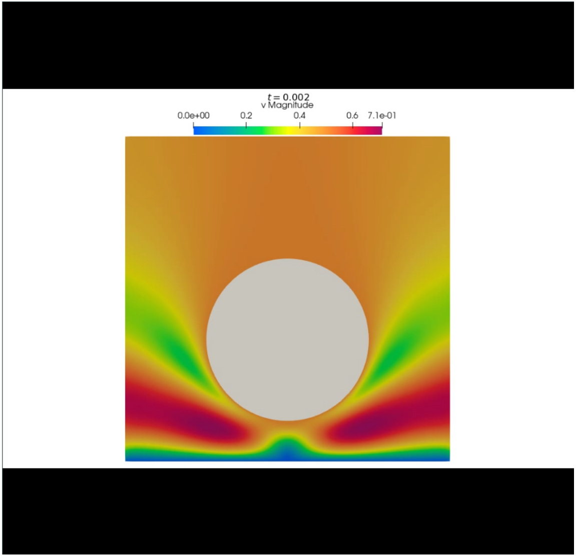
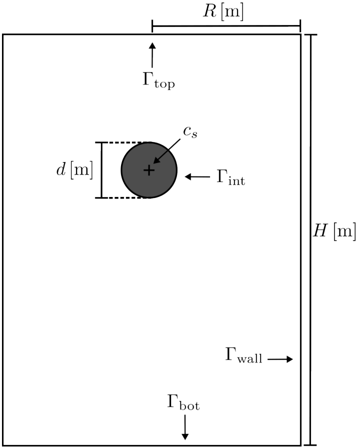


Figure 8: Total ball energy

Rebound



Benchmark



- Stefan Frei
- Karel Tůma
- Thomas Wick