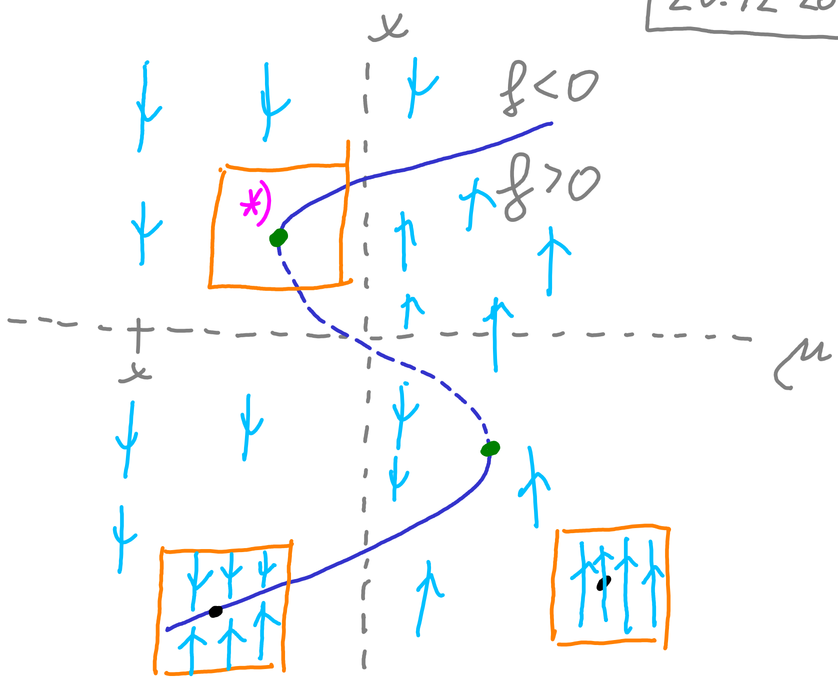
20.12.2020

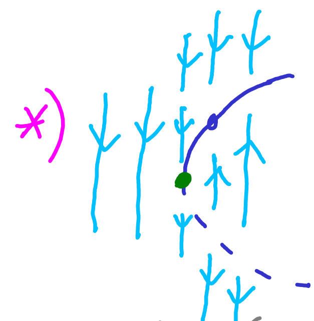
Př.: $x' = \underbrace{\mu + x - x^3}_{f(x,\mu)}$

stac. body: $\boxed{f(x,\mu)=0}$

$$\mu = x^3 - x$$

nutné podm. bif. $\rightarrow$ $\boxed{\frac{\partial f}{\partial x}(x,\mu)=0}$





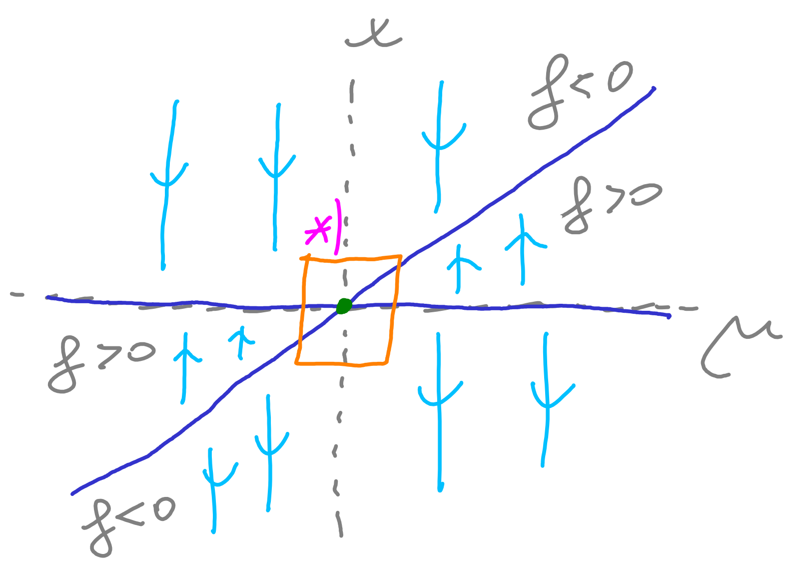
$(x_0,\mu_0) = \left(\frac{1}{\sqrt{3}}, \frac{1}{3} - \frac{1}{\sqrt{3}}\right)$

typ: sedlo-uzel

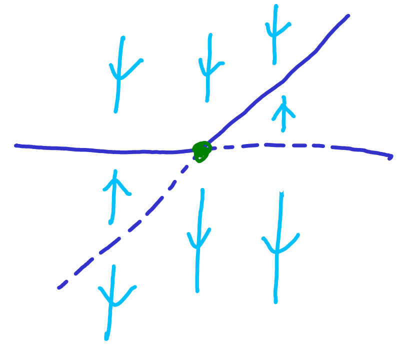
Př. 2 [transkritická bif.]

$x' = \underbrace{\mu x - x^2}_{= f(x,\mu)}$

$$\left.\begin{aligned} f &= x(x-\mu) = 0 \\ \frac{\partial f}{\partial x} &= \mu - 2x = 0 \end{aligned}\right]$$

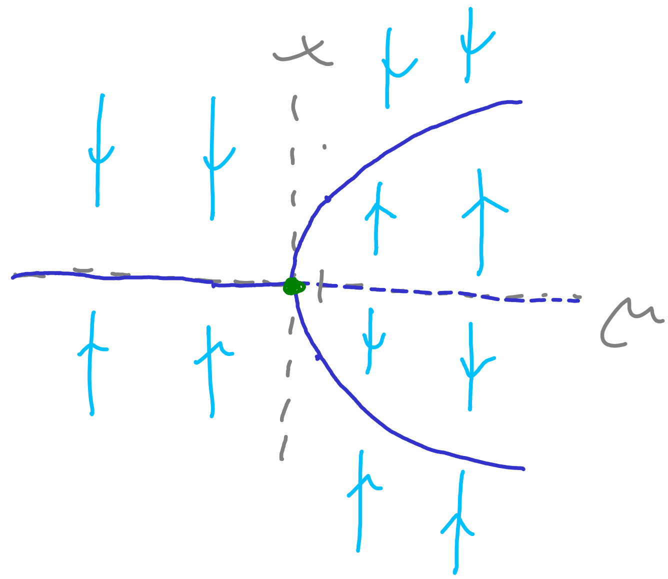


*)



Př. 3 [vidličková bif.]

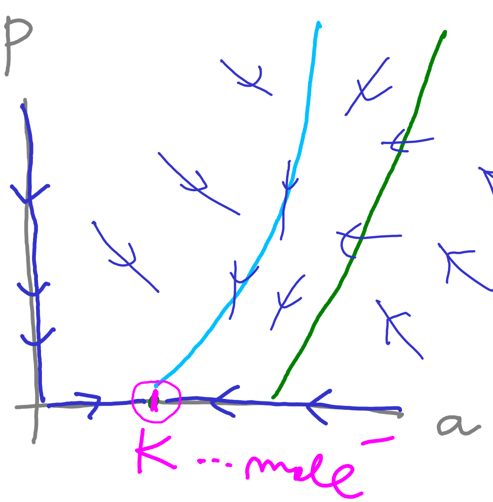
$$x' = \mu x - x^3 = x(\mu - x^2)$$



Příkl. 5 [sedlo-uzel v $\mathbb{R}^2$]

$$a' = a(K-a) + \frac{ap}{p+1} \qquad \text{(hmyz)}$$

$$p' = -\frac{p}{2} + \underbrace{\frac{ap}{p+1}}_{\text{symbióza}} \qquad \text{(rostlina)}$$



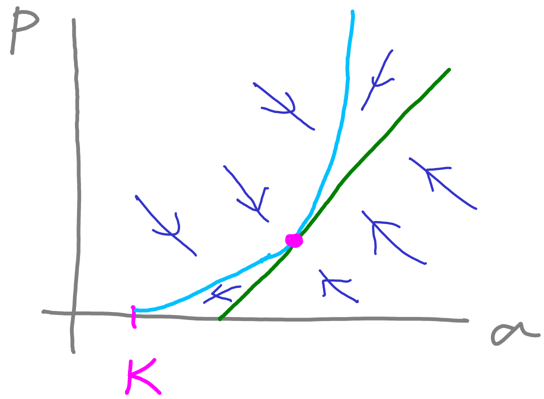
$$a' = 0 \iff p = \frac{a-K}{K+1-a} \quad (*)$$

$$p' = 0 \iff p = 2a-1 \quad (*)$$

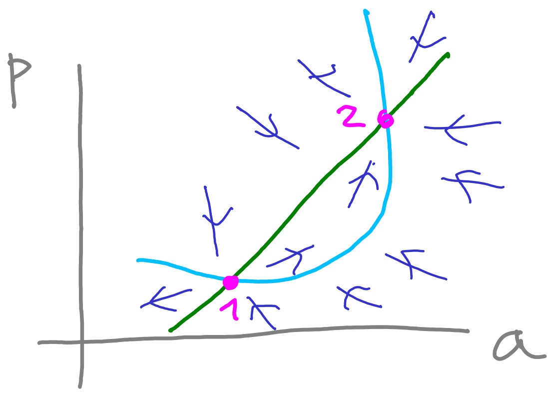
bifurkace: $K = K_0 = \sqrt{2}-1 \iff$ průnik (dotek)

$(*) \cap (*)$

$\sigma(A_0) = \{0, -\lambda_0\}$



$K > K_0 \Rightarrow$ sedlo & uzel!!



„uzel"

$\sigma(A_2) = \{-\varepsilon, -\tilde{\lambda}_0\}$

$\sigma(A_1) = \{+\varepsilon, -\tilde{\lambda}_0\}$

„sedlo"

7.1.2021

Příkl.

$$x' = -\mu x - y$$
$$y' = x + y^3$$

$f = 0$

$g = y^3$

$$A_\mu = \begin{pmatrix} -\mu, & -1 \\ 1, & 0 \end{pmatrix}$$

$$\sigma(\mu) = \left\{ -\frac{\mu}{2} \pm i\sqrt{4 - \frac{\mu^2}{4}} \right\}$$ μ malé

tj: $\alpha(\mu) = -\frac{1}{2}\mu$

$\omega(\mu) = \sqrt{4 - \frac{\mu^2}{4}}$

V. 19.4. $\Longrightarrow$ $\exists$ netriv. per. řešení v okolí $(0,0)$, pro μ malé

lze vidět více!

Věta [normální forma Hopf. bif.]

Nechť platí předpoklady Věty 19.4., nechť matice $A_0 = \begin{pmatrix} 0, & -\omega_0 \\ \omega_0, & 0 \end{pmatrix}$.

Potom $\exists$ hladká, nelineární záměna souř. t.ž. přív. rce má tvar $r' = r(d\mu + ar^2) + \dots$ kde $d = \alpha'(0)$, a lze spočíst ze vzorečku:

$$16a = f_{xxx} + f_{xyy} + g_{xxy} + \boxed{g_{yyy}} + \frac{1}{\omega_0}\Big[f_{xy}(f_{xx} + g_{xx}) - g_{xy}(g_{xx} + g_{yy}) - f_{xx}g_{xx} - f_{yy}g_{yy} \Big]\Big|_{(x,y,\mu)=(0,0,0)}$$

Pozn.: obecně: $r' = \underbrace{\overset{d\mu}{\lambda(\mu)}}\, r + \cancel{c_1 r^2} + c_2 r^3 + \dots$

$\dots (\mu \dots)$

aplikace:

$$\begin{aligned} x' &= \boxed{-\mu x - y} \\ y' &= \boxed{x} + \boxed{y^3} \end{aligned}$$

$f = 0$

$g = y^3$

$$A_\mu = \begin{pmatrix} -\mu, & -1 \\ 1, & 0 \end{pmatrix}$$

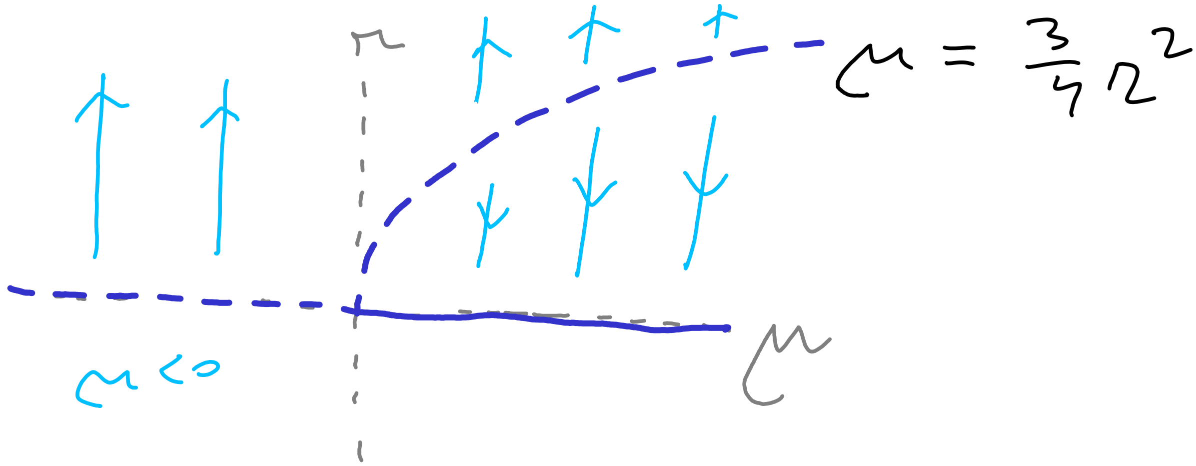
$$\sigma(\mu) = \left\{ -\frac{\mu}{2} \pm i\sqrt{4 - \frac{\mu^2}{4}} \right\} \quad \mu \text{ malé}$$

$$A_0 = \begin{pmatrix} 0, & -1 \\ 1, & 0 \end{pmatrix}; \text{ tj. } \omega_0 = \omega(0) = 1$$

$$d = \alpha'(0) = -\frac{1}{2}$$

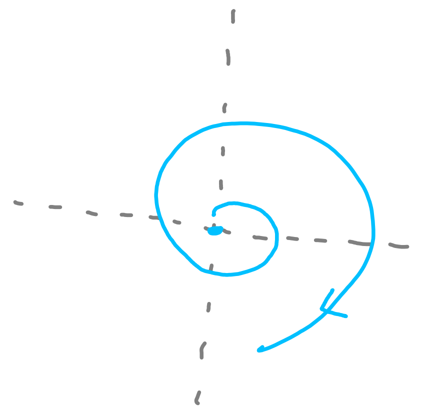
$16a = 6$; tj. $a = \frac{6}{16} = \frac{3}{8}$

$$\Longrightarrow \boxed{r' = r\left(-\frac{\mu}{2} + \frac{3}{8} r^2\right)} + \dots$$

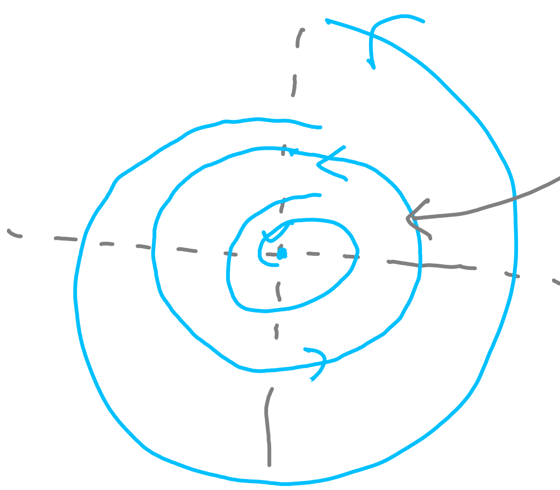


kartézské souř. ??

$\mu < 0$:



$\mu > 0$:



nestab. per. řešení