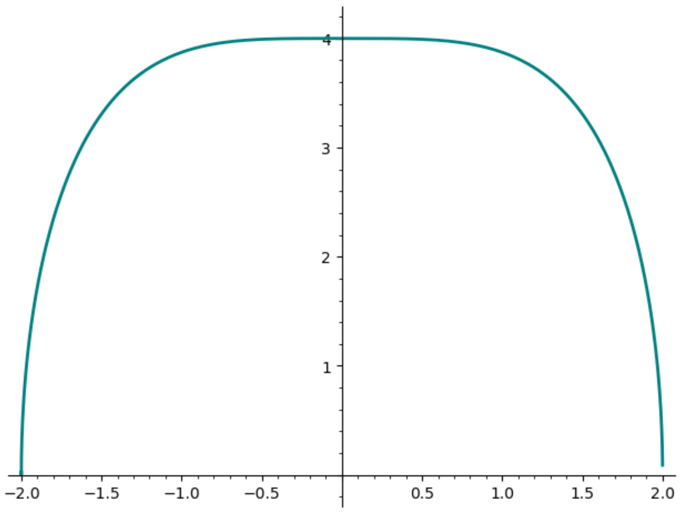
① $f(x)=\sqrt{16-x^4}$

(bez konvexity?)

$D_f=[-2,2]$, sudá

$f'(x)=-\frac{2x^3}{\sqrt{16-x^4}}$, $x\in(-2,2)$

$f'_{\pm}(\mp 2)=\pm\infty$



max: $f(0)=4$

min: $f(\pm 2)=0$

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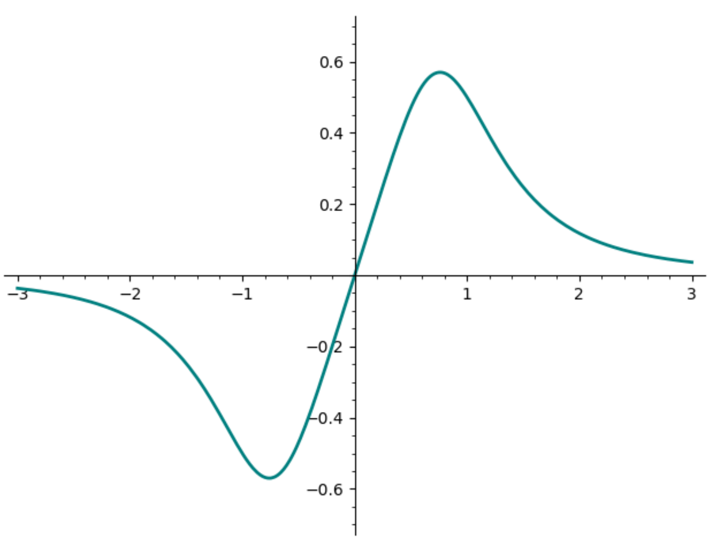
② $f(x)=\frac{x}{1+x^4}$ lichá, $f(x)\to 0$, $x\to\pm\infty$.

$f'(x)=\frac{1-3x^4}{(1+x^4)^2}=g(x^4)$, kde $g(y)=\frac{1-3y}{(1+y)^2}$

$f''(x)=g'(x^4)\cdot 4x^3=\frac{4x^3(3x^4-5)}{(1+x^4)^3}$



max/min: $x=\frac{\pm 1}{\sqrt[4]{3}}$

inflexe: $x=0,\ \pm\sqrt[4]{\frac{5}{3}}$

③ $f(x) = \ln\left(x^2 + x\right)$

$\mathcal{D}_f = (-\infty, 0) \cup (-1, +\infty)$

symetrie vůči $x = -\frac{1}{2}$

$\left(\text{neb: } x^2 + x = \left(x + \frac{1}{2}\right)^2 + \frac{3}{4}\right)$

$f(x) \to +\infty, \; x \to \pm\infty$

$\to -\infty, \; x \to 0-, 1+$

$$f'(x) = \frac{2x+1}{x(x+1)} \lessgtr 0 \quad \text{pro } x < 0 \text{ resp. } x > 1$$

$$f''(x) = -\frac{2x^2+2x+1}{x^2(x+1)^2} < 0 \quad \forall x \in \mathcal{D}_f$$

max/min $\nexists$

inflexe $\nexists$