Besselův integrál

$$J_n(x) = \frac{1}{2\pi} \int_{-\pi}^{\pi} \cos(nt - x\sin t)\, dt$$

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trik: $$\exp\left(+\frac{x}{2} z\right) = \sum_{m=0}^{\infty} \frac{1}{m!} \left(\frac{x}{2}\right)^m z^m$$ $z \neq 0$ konv. abs.

$$\exp\left(-\frac{x}{2}\cdot\frac{1}{z}\right) = \sum_{\lambda=0}^{\infty} \frac{1}{\lambda!}\left(-\frac{x}{2}\right)^{\lambda} z^{-\lambda}$$

Cauchyho součin řad:

$$\exp\left(\frac{x}{2}\left(z - \frac{1}{z}\right)\right) = \left(\sum_{m\geq 0}\right)\cdot\left(\sum_{\lambda\geq 0}\right)$$

vytvořující funkce.

$$= \sum_{n\in\mathbb{Z}} c_n \cdot z^n \quad , \quad \forall z \neq 0.$$

$$c_n = \sum_{\substack{m,\lambda\geq 0 \\ m-\lambda = n}} \frac{1}{m!}\frac{1}{\lambda!}\left(\frac{x}{2}\right)^m \left(-\frac{x}{2}\right)^{\lambda}$$

Klíčové pozorování: $c_n = J_n(x)$. $n \geq 0$ celé!

$$c_n = \sum_{\substack{m,\lambda\geq 0 \\ m-\lambda=n \\ m = \lambda+n}} \frac{1}{m!}\left(\frac{x}{2}\right)^m \frac{1}{\lambda!}\left(-\frac{x}{2}\right)^{\lambda} = \sum_{\substack{\lambda=0; \\ m=\lambda+n}}^{\infty} \frac{1}{\lambda!}\cdot\frac{1}{(\lambda+n)!}(-1)^{\lambda}\left(\frac{x}{2}\right)^{2\lambda+n}$$

$(\lambda+n)! = \Gamma(\lambda+n+1)$

$$= \left(\frac{x}{2}\right)^n \sum \quad = J_n(x).$$

$$f(z) = \exp\left(\frac{x}{2}\left(z - \frac{1}{z}\right)\right) = \sum_{n \in \mathbb{Z}} J_n(x)\, z^n \quad ; \quad z \neq 0.$$

Laurentovy koeficienty

$f(z) \in \mathcal{H}(P(0))$.

$$J_n(t) = \frac{1}{2\pi i} \int_{\varphi} \frac{f(z)}{z^{n+1}}\, dz \qquad \varphi(t) = e^{it}; \ t \in (-\pi, \pi).$$

$dz = i e^{it} dt$

$$= \frac{1}{2\pi i} \int_{-\pi}^{\pi} \exp\left(\frac{x}{2}\left(e^{it} - e^{-it}\right)\right) \cdot e^{-int}\, i\, dt$$

$$= \frac{1}{2\pi} \int_{-\pi}^{\pi} \exp\left(x \cdot i \sin t - int\right) dt$$

$\cos(\omega) + i \sin \omega; \ \omega = x \cdot \sin t - nt.$

$$= \mathrm{Re}\left(\qquad \right)$$