(1) $$\int_{-\infty}^{\infty} \frac{1+\cos x}{(x-\pi)(x^2+a^2)}\,dx; \qquad a>0.$$

návod: $g(z) = \dfrac{1+e^{iz}}{\dots\dots}$

(2) $$\left[\frac{1}{(x-i)^2(x^2+2x+2)}\right]^{\wedge}(\xi) =$$

(3) $$f(z) = \frac{1}{z^2(e^z+e^{\lambda})}; \qquad \lambda\in\mathbb{C}.$$

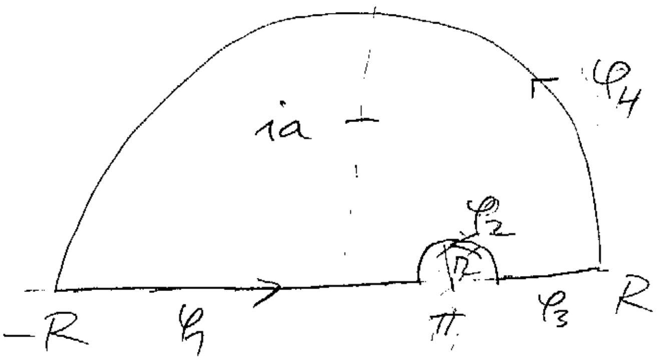
~~v bodě~~ (a) ~~$z_0 = 0$~~ ~~za~~

- typ singularity
- residuum

ve všech singularitách v $\mathbb{C}$

① $I = \int_{-\infty}^{\infty} \frac{1+\cos x}{(x-\pi)(x^2+a^2)}\,dx$; $a>0$.

$$g(z) = \frac{1+e^{iz}}{(z-\pi)(z^2+a^2)} \;;$$



$\varphi := \varphi_1 \dotplus \varphi_2 \dotplus \varphi_3 \dotplus \varphi_4$

[1]

$$\int_{\varphi_1} + \int_{\varphi_3} = \int_{I_{r,R}} \frac{\overbrace{1+e^{it}}^{=1+\cos t+i\sin t}}{(t-\pi)(t^2+a^2)}\,dt \;;$$

$I_{r,R} = (-R,R) \setminus U(\pi,r)$.

$$\mathrm{Re}\left\{\int_{\varphi_1} + \int_{\varphi_3}\right\} \to I$$

[1]

$$\int_{\varphi_4} \to 0 \;; \text{ neboť } |g(z)| \le \frac{1+\overbrace{|e^{iz}|}^{\le 1}}{(|z|-\pi)(|z|^2-a^2)} \sim \frac{1}{|z|^3}$$

L. o velké půlkružnici.

[2]

$$\int_{\varphi_2} \to i\pi\cdot A \;; \text{ kde } A = \lim_{z\to\pi} (z-\pi)\cdot g(z)$$

$$= \lim_{z\to\pi} \frac{1+e^{iz}}{z^2+a^2}$$

$$= 0 \quad (\text{limita} = \text{hodnota!!})$$

[2]

pr. velké $\displaystyle\int_{\varphi} g = 2\pi i \cdot \mathrm{res}_{ia}\, g$; R velké!

$$\Rightarrow \quad I = \mathrm{Re}\left\{ 2\pi i \cdot \mathrm{res}_{ia}\, g \right\}.$$

---

výpočet rezidua: $\displaystyle g(z) = \frac{1+e^{iz}}{(z-\pi)} \cdot \frac{1}{z^2+a^2}$;

$$\mathrm{res}_{ia} = \frac{1+e^{iz}}{z-\pi} \cdot \frac{1}{(z^2+a^2)'}\Bigg|_{z=ia}$$

$$= \frac{1+e^{-a}}{ia-\pi} \cdot \frac{1}{2ia} \, ; \quad ✓$$

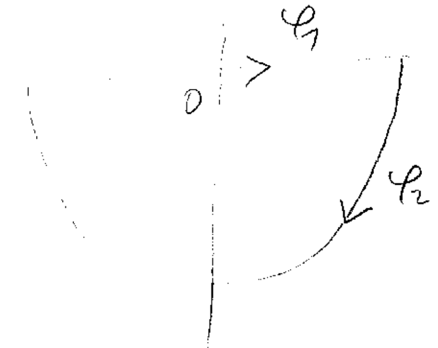
[2]

$$I = \mathrm{Re}\left\{ \frac{\pi(1+e^{-a})}{a} \cdot \frac{1}{ia-\pi} \right\} \qquad \frac{1}{ia-\pi} = \frac{-\pi-ia}{\pi^2+a^2}$$

$$= \frac{-\pi^2}{\pi^2+a^2} \cdot \frac{1+e^{-a}}{a} \, . \quad ✓$$

② $\left[\frac{1}{(x-i)^2(x^2+2x+2)}\right]^{\wedge}(\xi) =$

(a) $\xi \ge 0$:



$$g(z) = \frac{e^{-2\pi i \xi z}}{(z-i)^2(z^2+2z+2)};$$

singularity: $z_0 = i$

$z_{1,2} = -1 \pm i$

(a) $\varphi = \varphi_1 + \varphi_2$; $\int_{\varphi_1} g \to \hat{f}(\xi)$ [1]

$\int_{\varphi_2} g \to 0$; neboť $\left|e^{-2\pi i \xi z}\right| \le 1$ pro $\xi \ge 0$, $\operatorname{Im} z \le 0$

$$\left|(z-i)^2(z^2+2z+2)\right| \ge (|z|-1)^2(|z|^2-2|z|-2)$$

$\sim |z|^3$; tj.

$|g(z)| \le \frac{K}{|z|^3}$. L. o velké půlkružnici [2]

celkem: $-\hat{f}(\xi) = 2\pi i \cdot \operatorname{res}_{-1-i} g$;

$$\operatorname{res}_{-1-i} = \frac{e^{-2\pi i \xi(-1-i)}}{(1+2i)^2 \cdot (2(-1-i)+2)} = \frac{e^{-2\pi\xi} \cdot e^{2\pi i \xi}}{(4i-3)\cdot(-2i)}$$ [1]

$(1+2i)^2 = 1+4i-4 = 4i-3$ a $(8+6i)^{-1} = \frac{8-6i}{64+36} = \frac{8-6i}{100}$

$$-\hat{f}(\xi) = \frac{3+4i}{25}\pi \cdot e^{2\pi\xi(i-1)} \quad ; \quad \text{O.K. maple}$$

(b): $\xi < 0$: podobně: $\hat{f}(\xi) = 2\pi i \cdot \left(\operatorname{res}_i + \operatorname{res}_{-1+i} \right)$.

$$(\alpha)\ \underline{i}: \quad \operatorname{res}_i g = \left. \left(\frac{e^{-2\pi i \xi z}}{z^2+2z+2} \right)' \right|_{z=i}$$

$$= \frac{1}{(z^2+2z+2)^2} \cdot \left[e^{-2\pi i \xi z} \cdot (-2\pi i \xi)(z^2+2z+1) - e^{-2\pi i \xi z} \cdot (2z+2) \right] \Bigg|_{z=i}$$

$$\frac{1}{(1+2i)^2} \cdot e^{2\pi\xi} \cdot \left(2\pi\xi(1+2i) - (2i+2) \right)$$

$$= e^{2\pi\xi} \cdot \frac{-4-2i}{25} (5\pi\xi - 1 - 3i). \quad \text{maple. o.k.} \quad [\cancel{1}]^{2}$$

$$(\beta)\ -1+i: \quad \operatorname{res}_{-1+i} = \left. \frac{e^{-2\pi i \xi z}}{(z-i)^2 (2z+2)} \right|_{z=-1+i} \quad [1]$$

$$= \frac{e^{2\pi\xi(1+i)}}{2i}.$$

$(-1+i)(-i) = \underline{i+1}$

$\xi = 0$: [1]

③ $f(z) = \dfrac{1}{z^2(e^z + e^\lambda)}$; $\lambda \in \mathbb{R}$

(a) $z_0 = 0$: 2-násobný pól; $(e^0 + e^\lambda \neq 0)$ [1]

$$\operatorname{res}_0 = \left(\frac{1}{e^z + e^\lambda}\right)'\Big|_{z=0} = \frac{-e^z}{(e^z + e^\lambda)^2}\Big|_{z=0} = \frac{-1}{(1+e^\lambda)^2}$$ [2]

(b) $e^z = -e^\lambda = e^{\lambda + i\pi}$;

$z_k = \lambda + (2k+1)i\pi$; $k \in \mathbb{Z}$. [1]

$$\operatorname{res}_{z_k} = \frac{1}{z^2(e^z + e^\lambda)'}\Big|_{z=z_k} = \frac{1}{z_k^2 \cdot (e^{z_k})} = \frac{-1}{z_k^2 \cdot e^\lambda}$$ [1]

jednoduchý pól.