

A. Spočítejte integrály:

1. $\int_{-\infty}^{+\infty} \frac{x dx}{(x^2+4x+13)^2}$

8. $\int_{-\infty}^{\infty} \frac{dx}{(x^2+1)^3}$

2. $\int_0^{+\infty} \frac{x^2+1}{x^4+1} dx$

9. $\int_{-\infty}^{\infty} \frac{x^2 dx}{x^4+6x^2+25}$

3. $\int_{-\infty}^{+\infty} \frac{dx}{(x^2+a^2)(x^2+b^2)}$

10. $\int_0^{\infty} \frac{x^6 dx}{(x^4+a^4)^2}$

4. * $\int_{-\infty}^{+\infty} \frac{dx}{(x^2+1)^n}$

11. $\int_0^{\infty} \frac{x^4 dx}{(a+bx^2)^4}$

5. $\int_{-\infty}^{\infty} \frac{x^2 dx}{(x^2+1)(x^2+9)}$

12. $\int_{-\infty}^{\infty} \frac{dx}{(x^2+a^2)(x^2+b^2)^2}$

6. $\int_{-\infty}^{\infty} \frac{x^2-x+2}{x^4+10x^2+9} dx$

13. $\int_0^{\infty} \frac{x^4+1}{x^6+1} dx$

7. $\int_{-\infty}^{\infty} \frac{dx}{x^2-2ix-2}$

14. $\int_{-\infty}^{\infty} \frac{x^2 dx}{(x^2+4ix-5)^2}$

B. Spočítejte integrály:

1. $\int_0^{+\infty} \frac{\cos x}{(x^2+a^2)^3} dx$

6. $\int_0^{\infty} \frac{\sin ax}{x(x^2+b^2)}$

2. $\int_{-\infty}^{+\infty} \frac{(x^3+5x) \sin x}{x^4+10x^2+9} dx$

7. $\int_0^{\infty} \frac{x^2-b^2}{x^2+b^2} \frac{\sin ax}{x} dx$

3. $\int_0^{+\infty} \frac{x \sin x}{(x^2+a^2)^2} dx$

8. $\int_0^{\infty} \frac{x-\sin x}{x^3(x^2+a^2)} dx$

4. $\int_{-\infty}^{+\infty} \frac{(x-1) \cos x}{x^2-4x+5}$

9. $\int_{-\infty}^{\infty} \frac{\cos x dx}{x^2-\pi^2/4}$

5. $\int_0^{\infty} \frac{1-\cos ax}{x^2} dx$

10. $\int_{-\infty}^{\infty} \frac{\sin x dx}{x(x-3\pi)}$

C. Spočítejte integrály:

1. $\int_0^{\pi} \frac{\cos^2 x}{1-a \sin^2 x} dx, \quad a \in (0, 1)$

5. $\int_0^{2\pi} \frac{dx}{5+3 \cos x}$

2. $\int_0^{2\pi} \frac{dx}{(a+b \cos^2 x)^2}, \quad a, b > 0$

6. $\int_0^{2\pi} \frac{\cos^2 x dx}{13+12 \cos x}$

3. $\int_0^{\pi} \frac{\cos^4 x}{1+\sin^2 x} dx$

7. $\int_0^{2\pi} \frac{dx}{13+12 \sin x}$

4. $\int_0^{\pi} \frac{\sin(kx)}{\sin x} dx, \quad k \in \mathbb{N}$

8. $\int_{-\pi}^{\pi} \frac{\sin^2 x dx}{1-2a \cos x+a^2}, \quad a > 1$

Výsledky.

A1. $-\pi/27$ A2. $\pi/\sqrt{2}$ A3. $\pi/(ab(a+b))$ A4. $\pi(2n-3)!!/(2^{n-1}(n-1)!)$ A5. $\pi/4$ A6. $\frac{5}{12}\pi$ A7. 0 A8. $3\pi/8$ A9. $\pi/4$ A10. $3\sqrt{2}\pi/(16a)$ A11. $\pi/(32ab^2\sqrt{ab})$ A12. $\pi(2b+a)/(2ab^3(a+b)^2)$ A13. $4\pi/3$ A14. 0

B1. $\pi e^{-a}(a^2+3a+3)/(16a^5)$ B2. $\pi(e^{-1}+e^{-3})/2$ B3. $\pi e^{-a}/(4a)$
B4. $-\pi e^{-1}(\sin(2)-\cos(2))$ B5. $a\pi/2$ B6. $\pi(1-\exp(-ab))/2b^2$
B7. $\pi(2\exp(-ab)-1)/2$ B8. $\pi(a^2-2a+2-2e^{-a})/(4a^4)$
B9. -2 B10. $-2/3$

C1. $\pi(1-\sqrt{1-a})/a$ C2. $\pi(2a^3+5a^2b+4ab^2+b^3)/(2(a+b)^{7/2}a^{3/2})$
C3. $2\pi(\sqrt{2}-5/4)$ C4. π pro k liché, 0 pro k sudé C5. $\pi/2$ C6. $13\pi/45$
C7. $2\pi/5$ C8. π/a^2