1. $$\int_{-\infty}^{\infty} \frac{x^6}{(x^4+5x^2+4)^2}\,dx = \quad ?$$

2. $$\left[\frac{1}{e^x+e^{-x}+e+\frac{1}{e}}\right]^{\wedge}(\xi) = \quad ?$$

3. (a) $\dfrac{1}{z^4+z^8}$ -- Laurentův rozvoj (celý) $z_0=0$

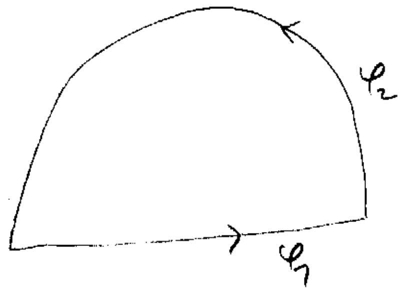
(b) $\dfrac{\sin z}{z^4+z^8}$ .. L. rozvoj (část) $z_0=0$

typ singularity

reziduum

① $g(z)=\dfrac{z^6}{(z^4+5z^2+4)^2}$; sing.: $(z^2+1)(z^2+4)$

$z=\pm i, \pm 2i$ **2**-násobné póly.



$$\int_{\varphi_1} g = \int_{-R}^{R} \frac{t^6}{(t^4+5t^2+4)^2}\,dt \to I;\quad R\to\infty.$$

$\int_{\varphi_2} g \to 0$; neboť $|g(z)| \le \dfrac{R^6}{(R^4-5R^2+4)^2} \sim \dfrac{1}{R^2}$;

$z\in\langle\varphi_2\rangle \Rightarrow |z|=R$, $\quad L(\varphi)=\pi R$.

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celkem: $$2\pi i\cdot\left(\operatorname{res}_i g + \operatorname{res}_{2i} g\right) = \int_\varphi g \longrightarrow I,\; R\to+\infty.$$

$$\operatorname{res}_i g = \left(\frac{z^6}{(z+i)^2(z^2+4)^2}\right)'\Bigg|_{z=i}$$

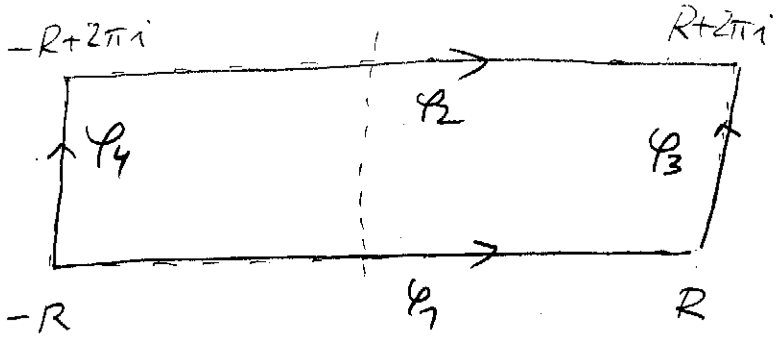
$$= \frac{1}{(z+i)^4(z^2+4)^4}\cdot\left\{6z^5(z+i)^2(z^2+4)^2 - z^6\left[2(z+i)(z^2+4)^2+(z+i)^2\cdot 2z\cdot 2\cdot(z^2+4)\right]\right\}\Bigg|_{z=i}$$

$$= -\frac{19i}{108};$$

$\operatorname{res}_{2i} g = \dfrac{2}{27} i$.

$$\boxed{I=\frac{11}{54}\pi}$$

(2) $g(z) = \dfrac{e^{-2\pi i \xi z}}{e^{z} + e^{-z} + e + \frac{1}{e}}$;



$\varphi := \varphi_1 \dotplus \varphi_3 \dot{-} \varphi_2 \dot{-} \varphi_4$;

$$\int_{\varphi_1} - \int_{\varphi_3} \to \hat{f}(\xi)\left(1 - e^{4\pi^2 \xi}\right).$$

omezené:

$$\int_{\varphi_4} g(z) = \int_0^{2\pi} \frac{e^{-2\pi i \xi(-R+2\pi i t)}}{\underbrace{e^{-R+2\pi i t}}_{\to 0} + \underbrace{e^{R-2\pi i t}}_{\to +\infty} + e + \frac{1}{e}}\, dt \to 0;\quad R \to +\infty.$$

podobně: $\int_{\varphi_3} g \to 0.$

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singularity: $e^{z} + e^{-z} + e + \frac{1}{e} = 0$; $\quad e^{z} = \lambda$

$\lambda + \frac{1}{\lambda} + e + \frac{1}{e} = 0$; $/ \cdot \lambda \quad (\lambda \neq 0 !!)$

$\lambda^2 + \lambda\left(e + \frac{1}{e}\right) + 1 = 0$

$(\lambda + e)\left(\lambda + \frac{1}{e}\right) = 0$; $\quad \lambda = -e$: $z = 1 + i\pi + 2k\pi i$

$\lambda = -\frac{1}{e}$: $z = -1 + i\pi + 2k\pi i$

$$\left.\begin{aligned} z_1 &= 1 + i\pi \\ z_2 &= -1 + i\pi \end{aligned}\right\} \in \operatorname{int} \varphi.$$

$$\operatorname{res}_{1+i\pi} g = \left.\frac{e^{-2\pi i \xi z}}{\left(e^{z}+e^{-z}+\frac{1}{e}+e\right)'}\right|_{z=1+i\pi} = \frac{e^{-2\pi i \xi(1+i\pi)}}{e^{1+i\pi}-e^{-1-i\pi}}$$

$$\left(e^{z}+e^{-z}+\frac{1}{e}+e\right)' = e^{z}-e^{-z}$$

$$= \frac{e^{2\pi^2\xi}\cdot e^{-2\pi i\xi}}{-e+\frac{1}{e}};$$

$$\operatorname{res}_{1-i\pi} g = \frac{e^{2\pi^2\xi}\cdot e^{2\pi i\xi}}{e-\frac{1}{e}};$$

$$\Rightarrow \hat{f}(\xi) = \frac{2\pi i}{1-e^{4\pi^2\xi}}\, e^{2\pi^2\xi}\cdot\left(\frac{e}{e^2-1}\right)\cdot\underbrace{\left[e^{2\pi i\xi}-e^{-2\pi i\xi}\right]}_{i\sin 2\pi\xi}$$

$$= \frac{2\pi e}{e^2-1}\cdot\frac{e^{2\pi^2\xi}}{e^{4\pi^2\xi}-1}\cdot\sin 2\pi\xi; \quad \xi\neq 0$$

$$\hat{f}(0) = (\text{re mozgitoli}): \frac{2\pi e}{e^2-1}\cdot\frac{2\pi}{4\pi^2} = \frac{e}{e^2-1}.$$

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3 (a) $\dfrac{1}{z^4+z^8} = \dfrac{1}{z^4}\cdot\dfrac{1}{1-(-z^4)} = \displaystyle\sum_{k=0}^{\infty}(-1)^k z^{4k-4}; \quad \forall |z|\in(0,1).$

(b) $$\frac{\sin z}{z^4+z^8} = \left(z-\frac{z^3}{6}+\frac{z^5}{5!}+\cdots\right)\cdot\left(\frac{1}{z^4}-1+z^4+\cdots\right)$$

$$= \frac{1}{z^3}+\frac{1}{z}\left(-\frac{1}{6}\right)+z\left(-1+\frac{1}{5!}\right)+\cdots$$

3-méroly pól;

$\operatorname{res} = -\dfrac{1}{6}.$