a) (rychlý výpočet): víme $\quad -\Delta \frac{1}{|x|} = 4\pi\,\delta_0$;

$$\mathcal{F}(-\Delta u) = 4\pi^2|\xi|^2\,\hat{u}$$

$$\mathcal{F}(\delta_0) = 1$$

tedy $\quad 4\pi^2|\xi|^2\left(\frac{1}{|x|}\right)^{\wedge} = 4\pi$

$$\left(\frac{1}{|x|}\right)^{\wedge} = \frac{1}{\pi|\xi|^2}.$$

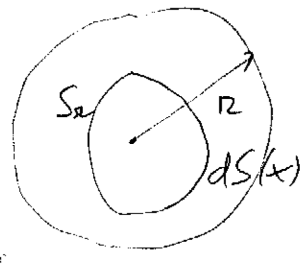
b) (přímý výpočet)

$$\langle \widehat{|x|^{-1}}, \varphi\rangle = \langle |x|^{-1}, \hat{\varphi}\rangle = \int_{\mathbb{R}^3} \frac{1}{|x|}\,\hat{\varphi}(x)\,dx = \lim_{R\to\infty} \int_{|x|<R} \frac{\hat{\varphi}(x)}{|x|}\,dx$$

$$= \lim_{R\to\infty} \int_{|x|<R} \left(\int_{\xi\in\mathbb{R}^3} \frac{e^{-2\pi i \xi\cdot x}}{|x|}\,\varphi(\xi)\,d\xi \right) dx$$

$$= \lim_{R\to\infty} \int_{\xi\in\mathbb{R}^3} \varphi(\xi)\cdot I(\xi)\,d\xi \,; \quad \text{kde } I(\xi) = \int_{|x|<R} \frac{e^{-2\pi i x\cdot\xi}}{|x|}\,dx.$$

„Fubiniho věta": $\quad \displaystyle\int_{|x|<R} = \int_0^R \left(\int_{S_r} dS(x) \right) dr$;



kde $\quad S_r = \{x\in\mathbb{R}^3;\ |x| = r\}$.

$$I(\xi) = \int_0^R \frac{1}{r}\left(\int_{S_r} e^{-2\pi i x\cdot\xi}\, dS(x)\right) dr$$

$$= \frac{2r}{|\xi|}\sin 2\pi r|\xi|$$

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$$\frac{2}{|\xi|}\int_0^R \sin 2\pi r|\xi|\, dr = \frac{2}{|\xi|}\,\frac{1-\cos(2\pi R|\xi|)}{2\pi|\xi|}$$

celkem:

$$\lim_{R\to\infty}\int_{\xi\in\mathbb{R}^3} \varphi(\xi)\,\frac{1}{\pi|\xi|^2} + \underbrace{\frac{\varphi(\xi)}{\pi|\xi|^2}\cos(2\pi R|\xi|)}_{\to\, 0}\, d\xi$$

$\to 0$ Riemann-Lebesgue lemma

neboť $\dfrac{\varphi(\xi)}{\pi|\xi|^2}\in L^1$.