Consider the equation

$$x' = f(t, x)$$

What type of symmetries (or invariances) of solutions can be deduced, provided there are some symmetries of the function $f(t, x)$?

(In the statements below, I and $\tilde{I}$ are open real intervals.)

1. Let $f(t, x)$ be T-periodic with respect to t, i.e. $f(t + T, x) = f(t, x)$. Then $x(t)$ is a solution on $I \iff x(t + T)$ is a solution on $\tilde{I} = \{t - T;\ t \in I\}$.

2. Let $f(t, x)$ be p-periodic with respect to x, i.e. $f(t, x + p) = f(t, x)$. Then $x(t)$ is a solution on $I \iff x(t) + p$ is a solution on I.

3. Let $f(t, -x) = -f(t, x)$, i.e. $f(t, x)$ is odd with respect to the variable x. Then $x(t)$ is a solution on $I \iff -x(t)$ is a solution on I.

4. Let $f(-t, x) = -f(t, x)$, i.e. $f(t, x)$ is odd with respect to the variable t. Then $x(t)$ is a solution on $I \iff x(-t)$ is a solution on $\tilde{I} = \{-t;\ t \in I\}$.

5. Let $f(-t, -x) = f(t, x)$, i.e. $f(t, x)$ is even with respect to (jointly) the variables t, x. Then $x(t)$ is a solution on $I \iff -x(-t)$ is a solution on $\tilde{I} = \{-t;\ t \in I\}$.