HW 2.1 Assume that the function $f(t, x) : \mathbb{R}^2 \to \mathbb{R}$ satisfies $f(-t, x) = -f(t, x)$, for all t. Prove that the following implications holds:
If $x(t)$, $t \in I$ is solution to $x' = f(t, x)$, then $\tilde{x}(t) := x(-t)$, $t \in \tilde{I}$ is also a solution to $x' = f(t, x)$, where $\tilde{I} = \{-t;\ t \in I\}$.
What kind of symmetry is this? (You can use this in the following problem.)

HW 2.2 Investigate the behavior of solutions to the first order ODE

$$x' = t(x - x^2)$$

without actually solving the equation. In particular, investigate monotonony and convexity of solutions as functions $x = x(t)$.
Sketch a picture (of size 10x10 cm at least) with examples of typical solutions.

HW 2.3 Investigate the behavior of solutions to the system

$$x' = x + y^3$$
$$y' = x - x^3$$

In particular, investigate the direction of solutions in the xy-plane.
Sketch a picture (of size 10x10 cm at least) with examples of typical solutions.

By the way, is there some symmetry here?

HW 2.4* Assume that $x = x(t)$ is a smooth positive function, defined on some interval $t \geq t_0$.
(i) Show that if $x' \leq c(1 + x)$ for some $c > 0$, there cannot be a blow-up for any finite time $t > t_0$.
(ii) Show that if $x' \geq cx^a$ with some $c > 0$ and $a > 1$, there indeed is a blow-up for some finite $t > t_0$.