HW1.1 Solve (=find the general solution) to the equation

$$x' = \sqrt[3]{x^2}t^3$$

In particular, find the solution with the initial condition $x(0) = -1$.

Note: $\sqrt[3]{x}$ is defined for all real x, and one has

$$\tfrac{d}{dx}\left(\sqrt[3]{x}\right) = \frac{1}{3\sqrt[3]{x^2}}$$

whenever $x \neq 0$.

HW1.2 Find a general solution to

$$x' + 4x = 2\sqrt{x}\exp t$$

Hint: Bernoulli.

HW1.3* Consider the equation

$$x' = \frac{x-t}{x+t}$$

(which we found unsolvable in the class). Solve by coming to polar coordinates. More precisely, write that

$$x(u) = r(u)\sin u$$
$$t(u) = r(u)\cos u$$

In other words, we assume that x, t and r (distance from the origin) are functions u (the polar angle). Differentiate and substitute to the original equation, using formally that

$$x' = \frac{dx}{dt} = \frac{dx/du}{dt/du}$$

Find the differential equation for $r = r(u)$. Solve and intepret geometrically.