Limity funkcí I

1. Dokažte z definice, že

   a) $\lim_{x\to 1}\left(\frac{x}{2}\right)^3=\frac{1}{8}$ b) $\lim_{x\to 1^+}[x]=1$ c) $\lim_{x\to 1^-}[x]=0$

   Spočtěte

2. (a) $\lim_{x\to 0}\frac{x^2-1}{2x^2-x-1}$ (b) $\lim_{x\to 1}\frac{x^2-1}{2x^2-x-1}$

3. $\lim_{x\to 2}\left(\frac{1}{x^2-2x}-\frac{x}{x^2-4}\right)$

4. $\lim_{x\to 0}\frac{(1+x)(1+2x)\dots(1+nx)-1}{x}$, $n\in\mathbb{N}$

5. $\lim_{x\to 1}\frac{x^{100}-2x+1}{x^{50}-2x+1}$

6. $\lim_{x\to 0}\frac{(1+mx)^n-(1+nx)^m}{x^2}$, $m,n\in\mathbb{N}$

7. $\lim_{x\to 1}\frac{x^{n+1}-(n+1)x+n}{(x-1)^2}$, $n\in\mathbb{N}$

8. $\lim_{x\to 1}\frac{x+x^2+\dots x^n-n}{x-1}$, $n\in\mathbb{N}$

9. $\lim_{x\to 1}\left(\frac{m}{1-x^m}-\frac{n}{1-x^n}\right)$, $m,n\in\mathbb{N}$

10. $\lim_{x\to 0}\frac{\frac{2}{x^2}+1}{\sqrt{\frac{3}{x^4}-\frac{6}{x^2}+5}}$

11. $\lim_{x\to 0+}\frac{(\sqrt{\frac{1}{x^2}+1}-\sqrt{\frac{1}{x^2}-1})}{x}$

12. $\lim_{x\to 0^+}\left(\sqrt{\frac{1}{x}+\sqrt{\frac{1}{x}+\sqrt{\frac{1}{x}}}}-\sqrt{\frac{1}{x}-\sqrt{\frac{1}{x}+\sqrt{\frac{1}{x}}}}\right)$

13. (a) $\lim_{x \to 16} \frac{\sqrt[4]{x} - 2}{\sqrt{x} - 4}$ (b) $\lim_{x \to 0} \frac{\sqrt{x+1} - 1}{x}$

14. $\lim_{x \to 0} \frac{\sqrt{1 - 2x - x^2} - (1 - x)}{x}$

15. $\lim_{x \to 0} \frac{\sqrt[3]{27 + x} - \sqrt[3]{27 - x}}{x + 2\sqrt[3]{x^4}}$

16. $\lim_{x \to 0} \frac{\sqrt[m]{1 + x} - \sqrt[n]{1 + x}}{x}$, $m, n \in \mathbb{N}$

17. $\lim_{x \to 0} \frac{\sqrt{1 + x} - \sqrt[3]{1 - x}}{\sqrt[3]{1 + x} - \sqrt{1 - x}}$

18. $\lim_{x \to a^+} \frac{\sqrt{x} - \sqrt{a} + \sqrt{x - a}}{\sqrt{x^2 - a^2}}$, $a \in \mathbb{R}_0^+$

19. $\lim_{x \to 0} \frac{\sqrt[m]{1 + ax}\,\sqrt[n]{1 + bx} - 1}{x}$, $m, n \in \mathbb{N}$, $a, b \in \mathbb{R}$