

Superseminar Week 01 (3.10.2024)

Repetition of secondary school mathematics

Solve the following inequalities in $\mathbb{R}$:

1. $\frac{x-2}{2x-8} \geq 1$

2. $\frac{x-1}{x+2} < \frac{x}{x+1} + 1$

3. $\log_{\frac{1}{3}}(x^2 - 3x + 3) \geq 0$

4. $|x^2 - 4x + 3| \leq |x^2 - 4|$

Logic, statements, predicates

1. Check the de Morgan's laws:

$$\overline{\alpha \vee \beta} \equiv \bar{\alpha} \wedge \bar{\beta}, \quad \overline{\alpha \wedge \beta} \equiv \bar{\alpha} \vee \bar{\beta}.$$

2. Using truth tables, prove the distribution laws:

$$\alpha \wedge (\beta \vee \gamma) \equiv (\alpha \wedge \beta) \vee (\alpha \wedge \gamma), \quad \alpha \vee (\beta \wedge \gamma) \equiv (\alpha \vee \beta) \wedge (\alpha \vee \gamma).$$

3. Express as simply as possible: $\forall \varepsilon > 0 \ \forall y \in \mathbb{R} : |y - 7| < 5 \Rightarrow |f(y) - 15| < \varepsilon$.