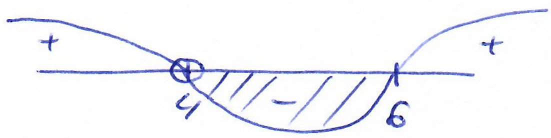
1. $$\frac{x-2}{2x-8} \geq 1$$

$$\frac{(x-2)-(2x-8)}{2x-8} \geq 0$$

$$\frac{x-2 \quad -2x+8}{2(x-4)} \geq 0$$

$$\frac{-x+6}{x-4} \geq 0$$

$$\frac{x-6}{x-4} \leq 0$$



Answer: $x \in (4, 6]$

2. $\frac{x-1}{x+2} < \frac{x}{x+1} + 1$

$$\frac{(x-1)(x+1) - x(x+2) - (x+1)(x+2)}{(x+1)(x+2)} < 0$$

$$\frac{x^2-1 - x^2 - 2x - (x^2+3x+2)}{(x+1)(x+2)} < 0$$

$$\frac{-2x-1 - x^2-3x-2}{(x+1)(x+2)} < 0$$

$$\frac{-x^2-5x-3}{(x+1)(x+2)} < 0$$

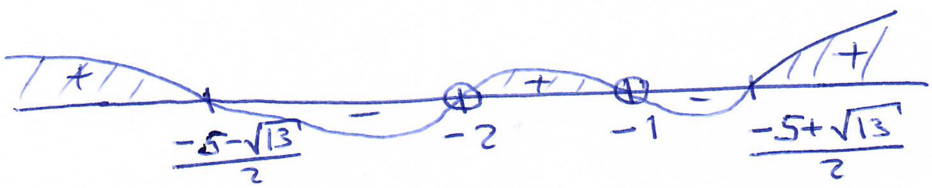
$$\frac{x^2+5x+3}{(x+1)(x+2)} > 0$$

Roots of the numerator: $x^2+5x+3=0$

$$x_{1,2} = \frac{-5 \pm \sqrt{25-12}}{2} = \frac{-5\pm\sqrt{13}}{2}$$

To draw $x_{1,2}$ on the axis, we need some estimates for them.
We have

$$3 < \sqrt{13} < 4 \qquad -4 < -\sqrt{13} < -3$$
$$-2 < \sqrt{13}-5 < -1 \qquad -9 < -5-\sqrt{13} < -8$$
$$-1 < \frac{\sqrt{13}-5}{2} < -\frac{1}{2} \qquad -\frac{9}{2} < \frac{-5-\sqrt{13}}{2} < -4$$



Answer:

$$x \in \left(-\infty, \frac{-5-\sqrt{13}}{2}\right) \cup (-2,-1) \cup \left(\frac{-5+\sqrt{13}}{2}, +\infty\right)$$

3. $\log_{\frac{1}{3}}(x^2-3x+3) \geq 0$

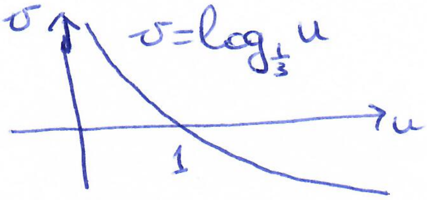
1) Domain for the function $v=\log_{\frac{1}{3}} u$ is $\{u: u>0\}$

i.e. $x^2-3x+3>0$

the discriminant for x^2-3x+3 is $D=9^2-12=-3<0$, hence no real roots and the coefficient in front of x^2 is $1>0$, hence the inequality $x^2-3x+3>0$, holds for all real x. Hence no restriction connected with domain of log

2) Function

$$v=\log_{\frac{1}{3}} u$$



is decreasing (since $\frac{1}{3}\in(0,1)$)

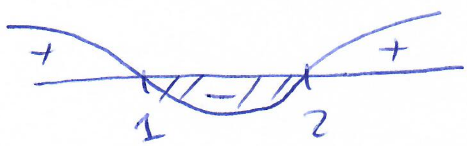
Hence when we take out the log, we need to change sign of the inequality

$$\log_{\frac{1}{3}}(x^2-3x+3) \geq \log_{\frac{1}{3}} 1 \quad (=0)$$

$$x^2-3x+3 \leq 1$$

$$x^2-3x+2 \leq 0$$

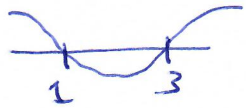
$$(x-2)(x-1) \leq 0$$



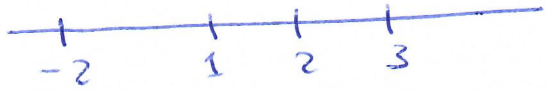
Answer: $x\in[1,2]$

4. $|x^2-4x+3| \le |x^2-4|$

We have $x^2-4x+3=(x-3)(x-1)$



$x^2-4=(x-2)(x+2)$



We will consider 5 different cases, to open the module.

1) $x \geq 3$

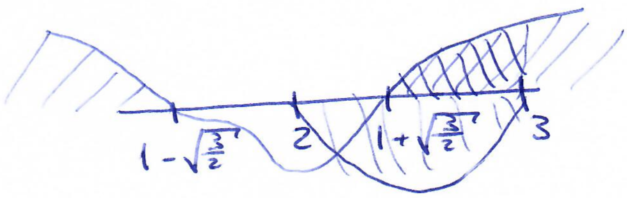
$x^2-4x+3 \le x^2-4;\quad -4x+7\le 0;\quad 4x\ge 7;\quad x\ge \frac{7}{4}$

We take intersection of $\{x: x\ge \frac{7}{4}\}$ with $\{x: x\ge 3\}$ and obtain $x \geq 3$

2) $2\le x<3$

$-x^2+4x-3 \le x^2-4;\quad 2x^2-4x-1\ge 0;\quad 2(x-1)^2 \geq 3$

$x \le 1-\sqrt{\frac{3}{2}}$ or $x \geq 1+\sqrt{\frac{3}{2}}$



We have $1<\sqrt{\frac{3}{2}}<2$,

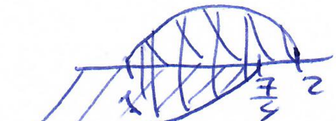
$2<1+\sqrt{\frac{3}{2}}<3,\quad -1<1-\sqrt{\frac{3}{2}}<0$

$\{x: x\in(-\infty, 1-\sqrt{\frac{3}{2}}] \cup [1+\sqrt{\frac{3}{2}}, +\infty)\} \cap \{x: x\in[2,3)\}$

$=\{x: x\in[1+\sqrt{\frac{3}{2}}, 3)\}$

3) $1\le x<2$

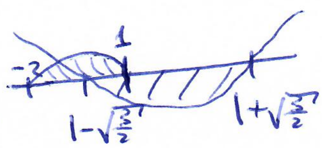
$-x^2+4x-3 \le -x^2+4;\quad 4x\le 7;\quad x\le\frac{7}{4}$



$\{x: 1\le x<2\} \cap \{x: x\le \frac{7}{4}\} = \{x: 1\le x\le \frac{7}{4}\}$

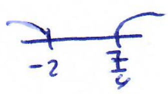
4) $-2\le x<1$

$x^2-4x+3 \le -x^2+4;\quad 2x^2-4x-1\le 0;\quad x\in[1-\sqrt{\frac{3}{2}}, 1+\sqrt{\frac{3}{2}}]$



$[1-\sqrt{\frac{3}{2}}, 1+\sqrt{\frac{3}{2}}] \cap [-2,1) = [1-\sqrt{\frac{3}{2}}, 1)$

5) $x<-2;\quad x^2-4x+3\le x^2-4;\quad 4x\ge 7;\quad x\ge\frac{7}{4};\quad [\frac{7}{4}, +\infty)\cap(-\infty,-2)=\emptyset$



Now we join intervals obtained in cases.

Answer: $\boxed{x\in[1-\sqrt{\frac{3}{2}}, \frac{7}{4}] \cup [1+\sqrt{\frac{3}{2}}, +\infty)}$

1. $\overline{\alpha \vee \beta} \equiv \overline{\alpha} \wedge \overline{\beta}$

α	β	$\overline{\alpha}$	$\overline{\beta}$	$\overline{\alpha} \wedge \overline{\beta}$	$\alpha \vee \beta$	$\overline{\alpha \vee \beta}$
1	1	0	0	0	1	0
1	0	0	1	0	1	0
0	1	1	0	0	1	0
0	0	1	1	1	0	1

$\overline{\alpha \wedge \beta} \equiv \overline{\alpha} \vee \overline{\beta}$

α	β	$\alpha \wedge \beta$	$\overline{\alpha \wedge \beta}$	$\overline{\alpha}$	$\overline{\beta}$	$\overline{\alpha} \vee \overline{\beta}$
1	1	1	0	0	0	0
1	0	0	1	0	1	1
0	1	0	1	1	0	1
0	0	0	1	1	1	1

2. $\alpha \wedge (\beta \vee \gamma) \equiv (\alpha \wedge \beta) \vee (\alpha \wedge \gamma)$

α	β	γ	$\beta \vee \gamma$	$\alpha \wedge (\beta \vee \gamma)$	$\alpha \wedge \beta$	$\alpha \wedge \gamma$	$(\alpha \wedge \beta) \vee (\alpha \wedge \gamma)$
1	1	1	1	1	1	1	1
1	1	0	1	1	1	0	1
1	0	1	1	1	0	1	1
1	0	0	0	0	0	0	0
0	1	1	1	0	0	0	0
0	1	0	1	0	0	0	0
0	0	1	1	0	0	0	0
0	0	0	0	0	0	0	0

$\alpha \vee (\beta \wedge \gamma) \equiv (\alpha \vee \beta) \wedge (\alpha \vee \gamma)$

α	β	γ	$\beta \wedge \gamma$	$\alpha \vee (\beta \wedge \gamma)$	$\alpha \vee \beta$	$\alpha \vee \gamma$	$(\alpha \vee \beta) \wedge (\alpha \vee \gamma)$
1	1	1	1	1	1	1	1
1	1	0	0	1	1	1	1
1	0	1	0	1	1	1	1
1	0	0	0	1	1	1	1
0	1	1	1	1	1	0	1
0	1	0	0	0	1	1	0
0	0	1	0	0	0	1	0
0	0	0	0	0	0	0	0

3. Simplify expression:

$$\forall \varepsilon > 0 \ \forall y \in \mathbb{R}: \ |y-7| < 5 \Rightarrow |f(y) - 15| < \varepsilon$$

Solution

We can write implication in terms of disjunction:

$$\alpha \Rightarrow \beta \quad \equiv \quad \beta \vee \bar{\alpha}$$

Hence,

$$|y-7| < 5 \Rightarrow |f(y) - 15| < \varepsilon$$

is the same as $\left(|f(y)-15| < \varepsilon\right) \vee \left(|y-7| \geq 5\right)$

Next, $|y-7| \geq 5$ is equivalent to $y \in (-\infty, 2] \cup [12, +\infty)$

Hence, the statement says, that for all $\varepsilon > 0$ and for all $y \in \mathbb{R}$, we have either $(y \in (-\infty, 2] \cup [12, +\infty))$ or $|f(y) - 15| < \varepsilon$.

For $y \in (-\infty, 2] \cup [12, +\infty)$ we cannot get any additional information.

But for $y \in (2, 12)$ we obtain, that $|f(y) - 15| < \varepsilon$.

This should be true for all $\varepsilon > 0$.

Hence, for any $y \in (2, 12)$, $f(y) = 15$.

[Indeed, to prove that, assume the contrary, that for some $y \in (2, 12)$, $f(y) \neq 15$. But then for the following choice of ε: $\varepsilon = \frac{1}{2}|f(y) - 15|$, the inequality $|f(y)-15| < \varepsilon$ becomes $|f(y) - 15| < \frac{1}{2}|f(y) - 15|$, which cannot be true.

This contradiction shows that indeed $f(y) = 15$]

Answer: $\forall y \in (2, 12): \ f(y) = 15$