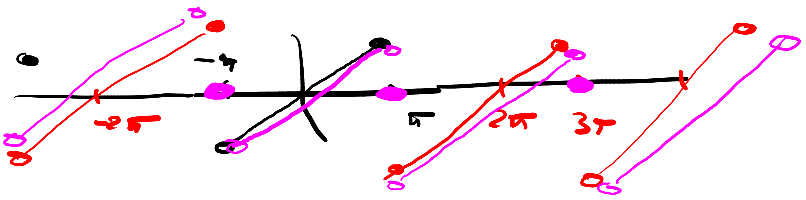
Fourierovy řady

Trig polynom $\quad a_k e^{ikx}$

$$a_0 + a_1 \cos x + b_2 \sin x + \\ a_2 \cos 2x \quad b_3 \sin 3x$$

$$\frac{a_0}{2} + \sum_{k=1}^{\infty} a_k \cos(kx) + b_k \sin(kx)$$

Př $\quad f(x) = x \quad [-\pi, \pi]$



$$a_k = \frac{1}{\pi} \int_{-\pi}^{\pi} x \cos(kx)\, dx \quad k \in \mathbb{N}_0$$

$$b_k = \frac{1}{\pi} \int_{-\pi}^{\pi} x \sin(kx)\, dx \quad k \in \mathbb{N}$$

$$a_0 = \frac{1}{\pi} \int_{-\pi}^{\pi} x \cdot 1\, dx = \frac{1}{\pi} \left[\frac{x^2}{2} \right]_{-\pi}^{\pi} = \frac{1}{\pi}\left(\frac{\pi^2}{2} - \frac{\pi^2}{2}\right) = 0$$

a_k : $\underbrace{x}_{l} \underbrace{\cos(kx)}_{s}$ lichá funkce $\to a_k = 0$

$$b_k = \frac{1}{\pi} \int_{-\pi}^{\pi} \underbrace{x}_{u} \underbrace{\sin(kx)}_{v' = \sin kx} dx = \frac{1}{\pi}\left[x \frac{-\cos kx}{k} \right]_{-\pi}^{\pi} - \frac{1}{\pi} \int_{-\pi}^{\pi} \frac{1}{k} \cos kx\, dx$$

$$u' = 1 \qquad v = \frac{-\cos kx}{k}$$

$$= \frac{1}{\pi}\left[x \frac{-\cos kx}{k} \right]_{-\pi}^{\pi} + \frac{1}{\pi} \left[\frac{1}{k} \frac{\sin kx}{k} \right]_{-\pi}^{\pi} = \frac{1}{\pi}\Big(\pi \frac{-\cos k\pi}{k} \\ - (-\pi) \frac{-\cos(-\pi k)}{k} \Big) \begin{matrix} +0 \\ -0 \end{matrix}$$

$$= -\frac{(-1)^k}{k} - \frac{(-1)^k}{k} = 2\frac{(-1)^{k+1}}{k}$$

$\cos(-k\pi) = \cos k\pi$

b_k

$$\mathcal{F}_f = \sum_{k=1}^{\infty} 2\frac{(-1)^{k+1}}{k} \sin(kx)$$

$\overset{?}{=} f(x)$ NE

1 0 -1 0 ...

$x = \frac{\pi}{2}$

$$f\left(\frac{\pi}{2}\right) = \sum_{k=1}^{\infty} \frac{2}{k}(-1)^{k+1} \sin k\frac{\pi}{2}$$

$$\frac{\pi}{2} = \sum_{k=1}^{\infty} 2\frac{(-1)^{k+1}}{2k-1}$$

$f \in BV([-\pi,\pi])$

$(V = 4\pi)$

$$\frac{\pi}{4} = \sum \frac{(-1)^{k+1}}{2k-1} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \dots$$

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konst, polynomy x^4

e^x sinh, cosh

sin, cos

sgn