Součty MŘ

- $\sum_{n=0}^{\infty} x^n = \frac{1}{1-x}$ $\qquad |x|<1$

- $f(x) = \sum a_n x^n$

  $f' = \sum a_n \cdot n \cdot x^{n-1}$

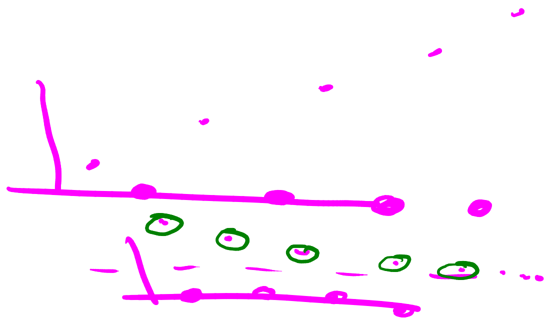
- $f(x) = \sum a_n x^n$

  $F(x) = \sum a_n \frac{x^{n+1}}{n+1}$

  $F'(x) = f$

$1 + 3x^2 + 5x^4 + 7x^6 + \dots$

- $\sum_{k=0}^{\infty} (2k+1) x^{2k} = f(x)$



- pol. konv. $\qquad \rho = \frac{1}{\limsup_{k\to\infty} \sqrt[k]{|a_k|}} = \frac{1}{1} = \underline{1}$:)

$$\begin{array}{ccccc} 1 & 0 & 3 & 0 & 5 \\ x^0 & x^1 & x^2 & x^3 & x^4 \end{array}$$

$a_k = k+1$

$j = 2k \qquad a_j = 2k+1 = j+1$

$\lim_{k\to\infty} \sqrt[k]{|k+1|} = 1$

$x \in (-1, 1)$

$\lim \sqrt[k]{k} = 1$

- $f(x) = \sum_{k=0}^{\infty} (2k+1) x^{2k}$ $\qquad \sum x^{2k} = \sum (x^2)^k$

$1 + 3x^2 \xrightarrow{\int} x + x^3$

$F(x) = \sum_{k=0}^{\infty} (2k+1) \frac{x^{2k+1}}{2k+1} = \sum_{k=0}^{\infty} \underbrace{x^{2k+1}}_{x + x^3} =$

$= x \sum_{k=0}^{\infty} x^{2k} = x \frac{1}{1-x^2}$

- $F' = f \qquad f = \left(\frac{x}{1-x^2}\right)' = \underline{\underline{\frac{1+x^2}{(1-x^2)^2}}}$

$x \in (-1, 1)$

$x = 1$ $\qquad$ $x = -1$

$\sum (2 \cdot 2+1)$ $\qquad$ $\sum (2 \cdot 2+1)$ $\qquad$ D (UP)

Závěr $\quad$ $f = \dfrac{1+x^2}{(1-x^2)^2}$ $\quad$ $x \in (-1, 1)$

$$\sum_{n=2}^{\infty} \frac{x^{n+1}}{(n-1)(n+1)} = f(x)$$

- $$\rho \qquad \frac{1}{\limsup \sqrt[n]{(n-1)(n+1)}} = 1$$

$x \in (-1, 1)$

Chceme $\frac{x^{n-1}}{n-1}$

- $$f'(x) = \sum_{n=2}^{\infty} \frac{x^n}{n-1}$$

$$= x \underbrace{\sum_{n=2}^{\infty} \frac{x^{n-1}}{n-1}}_{g(x)}$$

posun indexu $x \in (-1, 1)$

$$g'(x) = \sum_{n=2}^{\infty} x^{n-2} = \sum_{n=0}^{\infty} x^n = \frac{1}{1-x}$$

$x^0 + x^1 + x^2 \qquad x^0 + x^1 + \dots$

Integrujeme

$$g(x) = -\ln|1-x| + k = -\ln(1-x) + k$$

$$g(0) = -\ln(1-0) + k = k \longrightarrow k = 0$$

$$g(0) = \sum_{n=2}^{\infty} \frac{0^{n-1}}{n-1} = \frac{0^1}{1} + \frac{0^2}{2} + \dots = 0$$

- $$f'(x) = x(-\ln(1-x))$$

$\int$

$$f(x) = \frac{1}{4} x(x+2) - \frac{1}{2}(x^2-1)\ln(1-x)$$
$+ M$

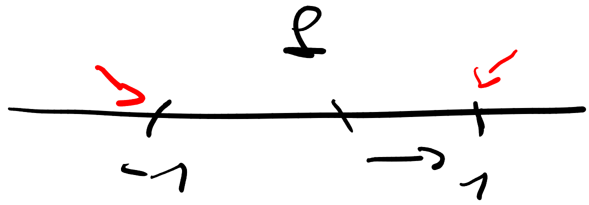
$\cdot\ f(0) = 0 + M \longrightarrow M = 0$

$$f(0) = \sum_{n=2} \frac{0^{n+1}}{(n+1)(n-1)} = 0 + 0 = 0$$

• Máme $x \in (-1,1)$

$$f(x) = \frac{1}{4} x(x+2) - \frac{1}{2}(x^2-1)\ln(1-x)$$

• $\displaystyle\sum_{n=2}^{\infty} \frac{x^{n+1}}{(n+1)(n-1)}$



$x = 1$

$$\sum \frac{1}{(n+1)(n-1)}$$

$x = -1$

$$\sum \frac{(-1)^{n+1}}{(n+1)(n-1)}$$

LSK $b_n = \frac{1}{n^2}$ AK $b_n = \frac{1}{n^2}$

?• $\displaystyle f(1) = \sum \frac{1^{n+1}}{(n+1)(n-1)} = \frac{3}{4}$

Abel

$$\sum \frac{1}{(n+1)(n-1)} = \lim_{x \to 1-} f(x)$$

$$= \lim_{x \to 1-} \frac{1}{4} x(x+2) - \frac{1}{2}\underbrace{(x^2-1)\ln(1-x)}_{0 \cdot (-\infty)} = \frac{3}{4} + 0 = \frac{3}{4}$$

$$f(-1) = \sum \frac{(-1)^{n+1}}{(n+1)(n-1)} = \lim_{x \to -1^+} f(x) =$$

$$= \lim_{x \to -1^+} \frac{1}{4}x(x+2) - \frac{1}{2}\underbrace{(x^2-1)}_{0}\underbrace{\ln(1-x)}_{\ln 2} = -\frac{1}{4} + 0 = \frac{-1}{4}$$

(spoj: $n \to -1$)

Závěr

$$\sum \frac{x^{n+1}}{(n-1)(n+1)} = f(x) \qquad x \in [-1, 1)$$

$$\sum \frac{1}{(n-1)(n+1)} = \frac{3}{4}$$