$$\sum_{n=1}^{\infty} x^n$$

$$f_n(x) = x^n$$

- bod. konv.

fix x: $\sum x^n$ k $\qquad x \in (-1,1)$

- stejnom. konv. $\qquad x \in (-1,1)$

fix n

$$r_n = \sup_{x \in (-1,1)} |x^n| = 1$$

$\sum r_n = \sum 1$ D :($\qquad$ Nevíme víc

- $[-k,k] \quad 0<k<1$

$$r_n = \sup_{x \in [-k,k]} |x^n| = k^n$$

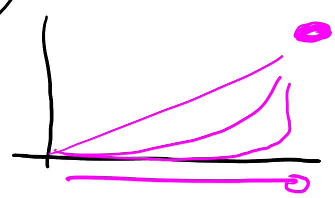
$\sum k^n$ konv. $\Rightarrow \sum x^n \rightrightarrows$ na $[-k,k]$

Závěr: $\sum x^n \overset{loc}{\rightrightarrows}$ na $(-1,1)$

? $\sum \rightrightarrows$ na $(-1,1)$ X

$f_n \rightrightarrows 0$ na $(-1,1)$?

? $x^n \rightrightarrows 0$? na $(-1,1)$ X



(2 týdny zpět $\longrightarrow$ NE)

Závěr: $\sum x^n \not\rightrightarrows$ na $(-1,1)$.

$$\sum_{n=1}^{\infty} \underbrace{\ln\left(1+\frac{2x}{x^2+n^2}\right)}_{f_n} \qquad x\in[0,\infty)$$

- bod. k.

$\ln(1+t)\le t \quad t>-1$

fix $x\in[0,\infty)$

$$\sum \ln\left(1+\frac{2x}{x^2+n^2}\right) \le \sum \frac{2x}{x^2+n^2} \le 2x\underbrace{\sum\frac{1}{n^2}}_{k} \;\; ☺$$

LSK $b_n = \frac{1}{n^2}$

$\sum f_n \;\; k \quad \forall x\in[0,\infty)$

- stejnom. na $[0,k]$

fix n

$$\sigma_n := \sup_{x\in[0,k]} \Big|\underbrace{\ln\left(1+\frac{2x}{x^2+n^2}\right)}_{\ge 0}\Big|$$

$$\ln\left(1+\frac{2x}{x^2+n^2}\right) \le \frac{2x}{x^2+n^2} \le \frac{2x}{n^2}$$

$$\le \frac{2k}{n^2}$$

$$\sum\frac{2k}{n^2} = 2k\sum\frac{1}{n^2}\,k \Rightarrow \sum f_n \rightrightarrows \text{ na } [0,k]$$

- závěr: $\sum f_n \overset{loc}{\rightrightarrows}$ na $[0,\infty)$

- $\sum f_n \rightrightarrows \cancel{[0,\infty)}$?

pevné N $\qquad x := N$

$$\sum_{n=N}^{2N} f_n(N) = \sum_{n=N}^{2N} \ln\left(1+\frac{2N}{N^2+n^2}\right) \geq$$

$$\geq \sum_{n=N}^{2N} \ln\left(1+\frac{2N}{N^2+(2N)^2}\right) \geq (N+1)\ln\left(1+\frac{2}{5N}\right)$$

$$\frac{2N}{5N^2}$$

$$\geq N \ln\left(1+\frac{2}{5N}\right) \geq \frac{1}{5} \qquad \longrightarrow \frac{2}{5}$$

$$\lim_{N\to\infty} \frac{N\ln\left(1+\frac{2}{5N}\right)}{\frac{2}{5N}} \cdot \frac{2}{5N} = \frac{2}{5}$$

$\exists \varepsilon = \frac{1}{5}$ $\qquad \forall n_0$ $\qquad n = N$ $\qquad x = N$

$m = 2N$

N zvoleno, aby

$$N\ln\left(1+\frac{2}{5N}\right) \geq \frac{1}{5}$$

BC podm. $\quad \sum \not\rightrightarrows$ na $[0,\infty)$

$$\sum \arctan \frac{2x}{x^2+n^3} \qquad x=0 \checkmark$$

x fix.

$$b_n = \sum \frac{2x}{x^2+n^3} \approx \sum \frac{1}{n^3}$$

$$\lim_{t\to 0} \frac{\arctan t}{t} = 1$$

LSE $\quad \frac{f_n}{b_n}$

$$\lim_{n\to\infty} \frac{\arctan \frac{2x}{x^2+n^3}}{\frac{2x}{x^2+n^3}} = 1$$