ODR n-tého řádu s PS

$$f = e^{\mu x}\left(P(x)\cos(\nu x) + Q(x)\sin(\nu x)\right)$$

$y'' + 9y = 8\cos x$ $\quad y(0) = 2 \quad y'(0) = 0$

- $y'' + 9y = 0$ $\quad y_H = c_1 e^{0x}\cos(3x) + c_2 e^{0x}\sin(3x)$

$\lambda^2 + 9 = 0$ $\quad x \in \mathbb{R} \quad c_1, c_2 \in \mathbb{R}$

$\lambda = 0 + 3i \quad \lambda = 0 - 3i$

- $8\cos x = e^{0x}(8\cos(1x) + 0 \cdot \sin(1x))$ (μ, $P(x)$, ν, $Q(x)$)

$\operatorname{st} P = 0 = \operatorname{st} Q$

$$y_P = x^0 e^{0x}\left(A\cos(1x) + B\sin(1x)\right)$$

$(\mu + i\nu) = 0 + 1i = i$

$\frac{\pm 3i}{}$ $m = 1$ // $\quad$ je i kořen? Není $\to m = 0$

$$y_P = A\cos x + B\sin x$$

$y_P' = -A\sin x + B\cos x$ $\quad y'' + 9y = 8\cos x$

$y_P'' = -A\cos x - B\sin x$

dosazení

$$\underline{-A\cos x} - \underline{B\sin x} + \underline{9A\cos x} + \underline{9B\sin x} = \underline{8\cos x} + 0\sin x$$

$\cos x:\quad -A + 9A = 8 \qquad A = 1$

$\sin x = \quad -B + 9B = 0 \qquad B = 0$

$y_P = 1\cos x \quad (+0\sin x)$

Dohromady

$$y = y_H + y_P = c_1\cos(3x) + c_2\sin(3x) + \cos x \qquad x, c_{1,2} \in \mathbb{R}$$

P.P. $\quad y(0) = 2 \quad y'(0) = 0$

$$y' = c_1(-\sin(3x)) \cdot 3 + c_2\cos(3x) \cdot 3 - \sin x$$

$2 = y(0) = c_1 \cdot 1 + c_2 \cdot 0 + 1 \qquad 0 = y'(0) = c_1 \cdot 0 + 3c_2 - 0$

$2 = c_1 + 1$ $\qquad$ $0 = 3c_2$ $\qquad$ $\underline{c_1 = 1 \quad c_2 = 0}$

Řešení S.P.P.

$$\underline{y = 1\cos 3x + \cos x} \qquad x \in \mathbb{R}$$

Pozn.

$$y'' + y = x + \sin x$$

$$\left.\begin{array}{ll} y'' + y = 0 & y_H \\ y'' + y = x & y_{P1} \\ y'' + y = \sin x & y_{P2} \end{array}\right\} y = y_H + y_{P1} + y_{P2}$$

$$y'' - 2y' + y = \frac{e^x}{x}$$

$x \neq 0$

$x \in (-\infty, 0)$ $\quad (0, \infty)$

- $y'' - 2y' + y = 0$ $\qquad y_H = c_1 e^{1x} + c_2 x e^{1x}$

  $\lambda^2 - 2\lambda + 1 = 0$

  $(\lambda - 1)^2 = 0$

  $\lambda = 1$ 2-nás.

- Variace konstant $\qquad y_p = c_1(x) e^x + c_2(x) x e^x$

$$y_p' = c_1'(x) e^x + c_1 e^x + c_2'(x) x e^x + c_2(e^x + x e^x)$$

Tedy $\qquad c_1' e^x + c_2' x e^x = 0$

$$y_p'' = c_1' e^x + c_1 e^x + c_2'(e^x + x e^x) + c_2(e^x + e^x + x e^x)$$

$$c_1' e^x + \cancel{c_1 e^x} + c_2'(e^x + x e^x) + c_2(\cancel{2e^x} + \cancel{x e^x}) - 2(\cancel{c_1 e^x} + c_2(\cancel{e^x} + \cancel{x e^x})) + \cancel{c_1 e^x} + c_2 \cancel{x e^x} = \frac{e^x}{x}$$

$$c_1' e^x + c_2' x e^x = 0$$

$$c_1' e^x + c_2'(e^x + x e^x) = \frac{e^x}{x}$$

$1 \cdot -1$

$$0 + 0 + c_2' e^x = \frac{e^x}{x}$$

$c_2' = \frac{1}{x}$ $\qquad c_2 = \ln|x| + k_1$

$c_1' e^x + \frac{1}{x} x e^x = 0$ $\qquad c_1' e^x = -e^x$ $\qquad c_1' = -1$ $\qquad c_1 = -x + k_2$

∘ řešení: $\quad y = (-x + k_2) e^x + (\ln|x| + k_1) x e^x$

$x \in (-\infty, 0)$

$x \in (0, \infty)$