Homogenní ODR $\quad y' = f\left(\frac{y}{x}\right)$

$z = \frac{y}{x} \rightarrow$ sep. prom.

Př. $\quad 2xyy' = 3y^2 - x^2$ $\quad \boxed{x \in \mathbb{R}}$

PP. $\quad y(2) = 6$ $\quad : x^2 !$ $\quad x \in (0,\infty)$ $\quad x \in (-\infty, 0)$

$$2\frac{y}{x}\,y' = 3\left(\frac{y}{x}\right)^2 - 1 \qquad y' = \frac{3\left(\frac{y}{x}\right)^2 - 1}{\frac{2y}{x}} \rightarrow f(z) = \frac{3z^2-1}{2z}$$

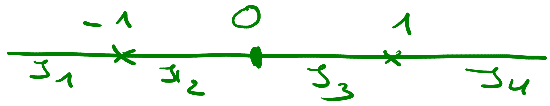
$z = \frac{y}{x} \rightarrow y = x \cdot z \qquad y' = z + xz'$

$$2z(z + xz') = 3z^2 - 1$$

$$2zz'x = z^2 - 1 \qquad z \equiv 0 \times \quad 0 = -1 \times$$

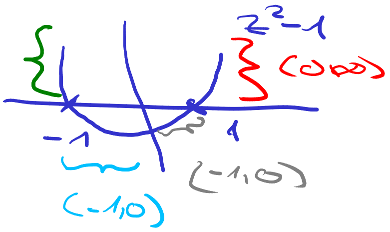
$$z' = \frac{z^2-1}{2z} \cdot \frac{1}{x} \qquad x \in I_1 = (-\infty, 0) \quad I_2 = (0,\infty)$$

$$\int \frac{2z}{z^2-1}\,dz = \int \frac{1}{x}\,dx \qquad z \equiv 1 \quad z \equiv -1 \quad \text{Řešení}$$



$$\ln|z^2-1| = \ln|x| + k$$

$G(y)$ $\qquad H(x)$



$G(-\infty,-1) = \mathbb{R} \qquad \forall x \in I_1, \forall x \in I_2$

$G(-1,0) = (-\infty, 0)$

$G(0,1) = (-\infty, 0)$

$G(1,\infty) = \mathbb{R} \qquad \forall x \in I_1, \forall x \in I_2$

$x \in (-\infty, 0)$ $\qquad \ln|x| + k < 0$

$$\ln|x| < -k$$

$$-x < e^{-k} \qquad \boxed{-e^{-k} < x} < 0$$

$x \in (0,\infty)$ $\qquad \ln|x| + k < 0$

$$\ln|x| < -k$$

$$\boxed{0 < x < e^{-k}}$$

$$\ln|z^2-1| = \ln|x| + k$$

$$|z^2-1| = e^k \cdot e^{\ln|x|}$$

$z \in J_1, J_4 \to (1,\infty)$ $+$

$(-\infty,-1)$ $-$

$$z^2-1 = e^t|x|$$
$$z^2 = 1+|x|e^t$$
$$z = \pm\sqrt{1+|x|e^t}$$

$z \in J_2, J_3$

$-$ $(-1,0)$ $\quad$ $(0,1)$ $+$

$$-z^2+1 = e^t|x|$$
$$1-e^t|x| = z^2$$
$$\pm\sqrt{1-e^t|x|} = z$$

$z \equiv \pm 1$

$x \in (-\infty,0)$

$x \in (0,\infty)$

$\longrightarrow y = x\cdot z$

$$y = \pm x\sqrt{1+|x|e^t}$$

$$y = \pm x \cdot 1$$

$\hookrightarrow x \in (-e^{-t},0)$

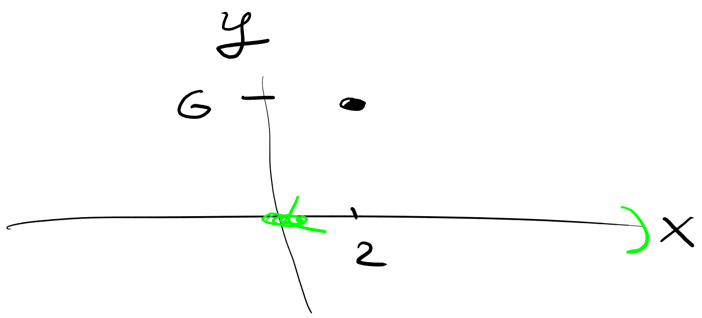
$(0,e^{-t})$

$$y = \pm x\sqrt{1-e^t|x|}$$

$x \in$

$y \neq \pm x$

DP $\quad y(2)=6$



$$y = x\sqrt{1+|x|e^t} = x\sqrt{1+|x|\cdot 4}$$

$$6 = 2\sqrt{1+2e^t}$$
$$9 = 1+2e^t$$
$$4 = e^t$$
$$\boxed{\ln 4 = t}$$

$$6 = 2\sqrt{1-e^t 2}$$
$$9 = 1-2e^t$$
$$-4 = e^t \quad :($$

Lepení:

$$y = \begin{cases} \pm x \\ \pm x\sqrt{1+|x|e^{x}} & x<0 \\ \pm x\sqrt{1-|x|e^{x}} & -e^{-1}<x<0 \\ x\sqrt{1+4x} & x \geq 0 \end{cases}$$

Ověření lepení: $\pm x\sqrt{1-xe^{x}}$ | $x\sqrt{1+4x}$, 0

Spoj.: $\lim\limits_{x\to 0-} \pm x\sqrt{1-xe^{x}} \overset{?}{=} \lim\limits_{x\to 0+} x\sqrt{1+4x}$

$0 = 0$ ✓

Der.: věta

$$\lim_{x\to 0-} \pm\left(\sqrt{1-xe^{x}} + x\,\frac{\frac{1}{2}\cdot(-e^{x}\ldots)}{\sqrt{1-xe^{x}}}\right) = \pm 1$$

$-1 \neq 1$

(zahodí 1 lepení)

$$\lim_{x\to 0+} \sqrt{1+4x} + x\,\frac{\frac{1}{2}\cdot 4}{\sqrt{1+4x}} = 1$$

Ověřit ODR:

$x=0$

$y=0$

$y'=1$

$2xyy' = 3y^2 - x^2$ ~ $\underline{\underline{0}}$

$0 = 0$ ✓

Lin ODR $\quad y' + p(x)y = q(x) \quad$ p, q spoj. na (a,b)

Př.: $$y' - \frac{y}{x} = x$$

$$p(x) = -\frac{1}{x} \qquad q(x) = x$$

spoj.: $x \in (-\infty, 0)$, $x \in (0, \infty)$

• $$y' - \frac{y}{x} = 0$$

$$y' = y \cdot \frac{1}{x}$$ • $y \equiv 0$

$$\int \frac{1}{y}\, dy = \int \frac{1}{x}\, dx$$

$$\ln|y| = \ln|x| + M$$

$$|y| = e^{M} \cdot e^{\ln|x|}$$

$$|y| = |x|\, L \qquad L > 0$$

$$y = \pm |x|\, L$$

$$y = x \cdot L$$

$L = 0$

$\underbrace{L > 0 \qquad L < 0}_{L \in \mathbb{R}}$

• $$y' - \frac{y}{x} = x$$

$y_p = x \cdot L(x)$ — neznámá fce

$$\cancel{L(x)} + xL'(x) - \cancel{\frac{xL(x)}{x}} = x$$

$$xL'(x) = x$$

$$L'(x) = 1$$

$$\boxed{L(x) = x + k}$$

• Celkem:

$$y = x(x + k) \qquad k \in \mathbb{R}$$

$x \in (-\infty, 0)$, $x \in (0, \infty)$