ODR

• $y' = x^2 + 1$ $\qquad y = \frac{x^3}{3} + x + k$

$y(0) = 3$ $\quad (x_0, \; y_0)$ $\qquad$ funkce difer. $\quad x \in I$ ot.

$$3 = y(0) = \frac{0^3}{3} + 0 + k \rightsquigarrow k = 3$$

$$y = \frac{x^3}{3} + x + 3 \qquad x \in \mathbb{R}$$

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so sep. prom. $\qquad y' = g(y) \cdot h(x)$

Př.: $y' = \frac{-y}{x}$ $\qquad x \neq 0$

$y' = y \cdot \frac{-1}{x}$

$g(y) = y \qquad h(x) = \frac{-1}{x}$

• x: $\quad I_1 = (-\infty, 0) \qquad I_2 = (0, \infty)$

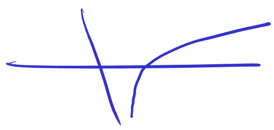
• y: $\quad g(y) \overset{?}{=} 0 \qquad y \equiv 0 \to x \in (-\infty, 0)$, $x \in (0, \infty)$

$0 = \frac{-0}{x}$

• y: $\quad J_1 = (-\infty, 0)$, $\quad J_2 = (0, \infty)$

• $y' = y \cdot \frac{-1}{x}$

$\frac{1}{y} y' = -\frac{1}{x}$ $\qquad y' = \frac{dy}{dx}$



$$\int \frac{1}{y}\, dy = \int -\frac{1}{x}\, dx$$

$$\ln|y| = -\ln|x| + k$$

• fix $I_1 = (-\infty, 0)$, $J_1 = (-\infty, 0)$, fix $k \in \mathbb{R}$

$G(J_1) = G(-\infty, 0) = \mathbb{R}$

pro které $x \in I_1$: $\quad -\ln|x| + k \in \mathbb{R}$ ✓ $\forall x \in (-\infty, 0)$

- $|y| = e^{-\ln|x| + k}$

$|y| = e^k \cdot e^{\ln|x|^{-1}}$

$y = \cancel{\pm} e^k \frac{1}{|x|}$

- $x \in (0, \infty)$

$I_1 = (-\infty, 0)$ $J_2 = (0, \infty)$

$G(J_2) = \mathbb{R}$ $y = + \frac{e^k}{|x|}$

$\underline{x \in I_1}$ $-\ln|x| + k \in \mathbb{R}$

- I_2 J_1

I_2 J_2

Záver: $y = \pm \frac{e^k}{|x|}$ $x > 0$, $x < 0$

$y \equiv 0$ — " —

Pr.: $y' = \sqrt[3]{y^2} \cdot 1$ - $x \in \mathbb{R} = I$

$\xrightarrow{g(y)}$ $h(x)$ $y(1) = 8$

- $g(y) = 0$ $y \equiv 0$ ✓ $x \in \mathbb{R}$

- y — 0 — $y \in (-\infty, 0) = J_1$ $(0, \infty) = J_2$

- $\int \frac{1}{\sqrt[3]{y^2}} \, dy = \int 1 \, dx$

$3 \sqrt[3]{y} = x + k$

$G(y)$ $H(x)$

- fix $x \in I = \mathbb{R}$, $y \in J_1 = (-\infty, 0)$, fix $k \in \mathbb{R}$

$G(J_1) = (-\infty, 0)$ $x + k \in (-\infty, 0)$

$x + k < 0$

$\underline{x < -k}$

• $\sqrt[3]{y} = \frac{x+2}{3}$

$$\boxed{y = \left(\frac{x+2}{3}\right)^3 \qquad x < -2}$$

• fix $x \in I$, $y \in J_2 = (0, \infty)$ fix 2

$G(J_2) = (0, \infty)$

$x + 2 > 0$

$y = \left(\frac{x+2}{3}\right)^3$ $\boxed{x > -2}$

• Řešení: chceme na $x \in \mathbb{R}$

(a) $y = 0$ $x \in \mathbb{R}$ $\times$

(b)

$$y = \begin{cases} \left(\frac{x+2}{3}\right)^3 & x < -2 \\ 0 & x \geq -2 \end{cases}$$

no sedí PP $\times$

(c)

$$y = \begin{cases} 0 & x < -2 \\ \left(\frac{x+2}{3}\right)^3 & x \geq -2 \end{cases}$$

(d)

$$y = \begin{cases} \left(\frac{x+2}{3}\right)^3 & x \leq -2 \\ 0 & -2 < x < -m \\ \left(\frac{x+m}{3}\right)^3 & x \geq -m \end{cases}$$

$-2 < -m$ $\boxed{m \leq 2}$

• slepení: spoj? v -2?

(b) $\lim_{x \to -2-} y \overset{?}{=} \lim_{x \to -2+} y$

$$\lim_{x\to -2-}\left(\frac{x+2}{3}\right)^3 = 0 \overset{\checkmark}{=} \lim_{x\to -2+} 0 = 0$$

dca ?:

$$y'_- \overset{?}{=} y'_+$$

$$\lim_{x\to -2-} y' \overset{?}{=} \lim_{x\to -2+} y'$$

$$\lim_{x\to -2-} 3\left(\frac{x+2}{3}\right)^2\cdot\frac{1}{3} = 0 \overset{\checkmark}{=} \lim_{x\to -2+} 0 = 0$$

dca în -2 $\exists$ (v.a. $= 0$)

} derivabil ✓

(c),(d) analog.