$\sum a_n$

- NP
- $\sum |a_n|$ $\longrightarrow$ Cauchy $\sqrt[n]{\ }$, d'Ale $\frac{a_{n+1}}{a_n}$, SK, LSK $\quad K \Rightarrow \sum a_k$
- $\sum a_n$ $\longrightarrow$ Leibniz "Lajbnic", Dirichlet, Abel

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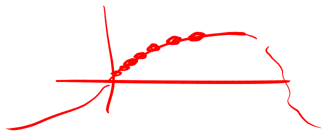
$\sum \frac{\sin(\overset{\to\infty}{n})}{n}$

Dirichlet

$\sum \frac{\sin(\overset{\to 0}{\frac{1}{n}})}{n}$



LSK $\frac{\frac{1}{n}}{n} = \frac{1}{n^2} = b_n$

$$\frac{\cancel{e^x+1}}{e^x-1}\cdot\frac{e^x(\cancel{e^x}+1)-(\cancel{e^x}-1)e^x}{(e^x+1)^{\cancel{2}}} =$$

$$= \frac{2e^x}{(e^x-1)(e^x+1)}$$

$$\sum \left|\arctan\left(\frac{e^n}{e^n+1}\right)\ln\left(\frac{e^n-1}{e^n+1}\right)\cos n\right| \quad Z :)$$

$$\le \sum \frac{\pi}{2}\cdot\left|\ln\left(\frac{e^n-1}{e^n+1}\right)\right| 1$$

$$\sum\left|\ln\left(\underbrace{\frac{e^n-1}{e^n+1}}_{<1}\right)\right| = \sum -\ln\left(\frac{e^n-1}{e^n+1}\right) \quad Z \checkmark$$

$$b_n = -\left(\frac{e^n-1}{e^n+1}-1\right) \qquad b_n = -\frac{e^n-1-e^n-1}{e^n+1} = \frac{+2}{e^n+1}$$

$$\lim_{n\to\infty}\frac{a_n}{b_n} = \lim \frac{-\ln\left(\frac{e^n-1}{e^n+1}\right)}{\frac{+2}{e^n+1}}$$

Heine $\quad x_n = \frac{e^n-1}{e^n+1} \xrightarrow{} t \qquad x_n \to 1$

$$\lim_{t\to 1}\frac{-\ln t}{-(t-1)} = +1 \in (0,\infty) \quad ZSZ$$

$$Z: \quad \sum \frac{2}{e^n+1} \le \sum \frac{2}{e^n} = 2\sum\left(\frac{1}{e}\right)^n$$

$Z :) \qquad Z :)$