Leibniz

$$\sum \underbrace{\cos(n\pi)}_{(-1)^n} \cdot \underbrace{\frac{n^2}{n^3+16}}_{b_n}$$

$$\lim_{n\to\infty} \underbrace{\frac{n^2}{n^3}}_{0} \cdot \frac{1}{\underbrace{1+16/n^3}_{1}} = 0$$

- $a_n \geq a_{n+1}$ $\qquad a_n - a_{n+1} \geq 0$ $\qquad \frac{a_n}{a_{n+1}} \geq 1$

- $f(x) = \frac{x^2}{x^3+16}$ $\qquad x > 0$

$$f'(x) = \frac{-x^4+32x}{(x^3+16)^2} \qquad \overset{>0}{x}(32-x^3)$$

pro $x > \sqrt[3]{32}$

$f' < 0$

pro $n \geq 4$ b_n klesající

Závěr: Leibniz: $\sum$ konverguje

$$\sum_{n=2}^{\infty} \frac{\sin(3n)}{\ln n}$$

$\sum \sin(3n)$ má ohr. č. súčty

$$\underbrace{\frac{1}{\ln n}}_{b_n} \qquad b_n \to 0$$

monotónne

Dirichlet $\to$ $\sum \frac{\sin(3n)}{\ln n}$ K

$$\sum_{n=2}^{\infty} \underbrace{\frac{\sin(3n)}{\ln n}}_{a_n} \cdot \underbrace{\frac{n-1}{n+1}}_{b_n}$$

z Abela Zadubk

$\sin(3n)$ $\qquad \frac{n-1}{(n+1)\ln n} \to$ ↗ :(

$\sum a_n$ K

b_n:

$$0 \le \frac{n-1}{n+1} \le 1 \checkmark \qquad \lim_{n\to\infty} \frac{n-1}{n+1} = 1$$

$b_n \nearrow ?$ $\qquad b_n \overset{?}{\le} b_{n+1} \checkmark$ $\qquad b_n$ neklesa. b_n om.

$$\frac{n-1}{n+1} \overset{?}{\le} \frac{(n+1)-1}{(n+1)+1}$$

$$(n-1)(n+2) \overset{?}{\le} n(n+1)$$

$$n^2+n-2 \le n^2+n$$

$$-2 \le 0 \quad \checkmark$$

Varování

$\sum \sin n^2$

$\sum \cos^2 n$

$\sum \sin(n + \ln n)$

n, m

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f klesající $\forall x_1 x_2 : x_1 < x_2$

$f(x_1) > f(x_2)$