$$\sum_{n=1}^{\infty} \frac{2^n}{n!}$$

$a_n > 0$

d'Al po délové

$$\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \lim \frac{\frac{2^{n+1}}{(n+1)!}}{\frac{2^n}{n!}} = \lim \frac{n!}{(n+1)!} \cdot \frac{2^{n+1}}{2^n}$$

$$= \lim_{n\to\infty} \frac{2}{n+1} = 0 < 1$$

Závěr: Zan k

$$\sum \frac{2^n}{n!}$$

$a_n \geq 0$

Cauchy $\sqrt{\ }$

$$\lim \sqrt[n]{a_n} = \lim \sqrt[n]{\frac{2^n}{n!}} = \lim \frac{2}{\underbrace{\sqrt[n]{n!}}_{\infty}} = 0 \underset{\checkmark}{<} 1$$

Závěr: $\sum a_n$ konv.

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$$\sum \frac{1}{n^2} \qquad \sum \frac{1}{n}$$

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$$\lim \sqrt[n]{4^n + a^n}$$

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$$\limsup \sqrt[n]{\frac{n^7}{2^n + 3^n}}$$

$$\leq \frac{\sqrt[n]{n^7}}{3\sqrt[n]{\left(\frac{2}{3}\right)^n + 1}} \underset{\checkmark}{\leq} \frac{1}{3} \frac{\sqrt[n]{n^7}}{\cancel{\sqrt[n]{\left(\frac{2}{3}\right)^n + 1}}}$$

$$= \frac{1}{2} \sqrt[n]{n^7} = \frac{1}{2} \cdot 1$$

$$\leq \frac{\sqrt[n]{n^7}}{\sqrt[n]{2^n}}$$

$$\geq \frac{\sqrt[n]{n!}}{n}$$

$$\lim \left(1 - \frac{2}{n+1}\right)^{n+1} = e^{-2}$$

$$\frac{\lim}{n+1-1-1}$$

$$\lim_{n\to\infty} \left(\frac{n-1}{n+1}\right)^{n-1} = \lim \left(1 + \frac{-2}{n+1}\right)^{n+1-2}$$

$$= \lim \underbrace{\left(1 - \frac{2}{n+1}\right)^{n+1}}_{e^{-2}} \cdot \underbrace{\left(1 + \frac{-2}{n+1}\right)^{-2}}_{1}$$

$$e^{(x-1)\ln\frac{x-1}{x+1}}$$

$$= \frac{1}{e^2} < 1$$

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$$\lim \sqrt[n]{\frac{n^2+1}{2^n-1}}$$

$$\frac{\sqrt[n]{n^2}}{\sqrt[n]{2^n}} \le \sqrt[n]{\frac{n^2+1}{2^n-1}} \le \frac{\sqrt[n]{n^2+n^2}}{\sqrt[n]{2^{n-1}}}$$

$$\downarrow$$

$$\frac{1}{2}$$

$$\le \frac{\sqrt[n]{2}\cdot\sqrt[n]{n^2}}{\sqrt[n]{2^n}\cdot\sqrt[n]{\frac{1}{2}}}$$

$$\frac{1}{2} \leftarrow$$