$$\lim_{x\to 0}\left(\frac{1+\operatorname{tg}x-\sin x\,(1-\cos x)}{1+\sin x}\right)^{\frac{1}{\underbrace{\operatorname{arcsinh}^4 x}_{(\operatorname{ash} x)}}} \rightsquigarrow$$

Počítali jsme po přís. na prim. fce

$$\lim_{x\to 0}\underbrace{\frac{1}{\operatorname{ash}^4 x}}_{\searrow 0}\ \ln\Bigg(\underbrace{\frac{1+\operatorname{tg}x-\sin x\,(1-\cos x)}{1+\sin x}}_{\to\ \ln 1=0}\Bigg) \rightsquigarrow$$

kvůli ln [známe $\frac{1}{y}\ln(1+y)\to 1,\ y\to 0$]

$$\lim_{x\to 0}\frac{1}{\operatorname{ash}^4 x}\ln\left(\frac{1+\sin x-\sin x+\operatorname{tg}x-\sin x\,(1-\cos x)}{1+\sin x}\right)$$

$$=\lim_{x\to 0}\frac{1}{\operatorname{ash}^4 x}\ln\Bigg(1+\overbrace{\frac{-\sin x+\operatorname{tg}x-\sin x\,(1-\cos x)}{1+\sin x}}^{y}\Bigg)\frac{y}{y}=$$

zpět k x [y jen pro krátkost]

$$=\lim_{x\to 0}\frac{1}{\operatorname{ash}^4 x}\left(\frac{-\sin x+\operatorname{tg}x-\sin x\,(1-\cos x)}{1+\sin x}\right)=$$

$$=\lim_{x\to 0}\frac{1}{\operatorname{ash}^4 x}\left(\frac{-\sin x+\frac{\sin x}{\cos x}-\sin x\,(1-\cos x)}{1+\sin x}\right)=$$

$$=\lim_{x\to 0}\frac{1}{\operatorname{ash}^4 x}\left[\frac{-\sin x\cos x+\sin x-\sin x\cos x\,(1-\cos x)}{(1+\sin x)\cos x}\right]$$

$$\overset{(?)}{=}\lim_{x\to 0}\frac{1}{\operatorname{ash}^4 x}\ \frac{-\sin x(\cos x-1+\cos x\,(1-\cos x))}{1\cdot 1}=$$

[x^4 dodám kvůli $1-\cos x$ & $1-\cos x$] → mohu dodat okolo dále

$$=\lim_{x\to 0}\frac{1}{\operatorname{ash}^4 x}\ \frac{-\sin x\,(1-\cos x)(-1+\cos x)\,x^4}{x^2\qquad x^2}=$$

VZOREC $\frac{1-\cos x}{x^2}\to\frac{1}{2},\ x\to 0$

$$=(-1)\frac{1}{2}\cdot\left(-\frac{1}{2}\right)\lim_{x\to 0}\left(\frac{x^4\sin x}{\operatorname{ash}^4 x}\right)\overset{\boxed{?}}{=}\frac{1}{4}\underbrace{\lim_{x\to 0}\sin x}_{0}\cdot\lim_{x\to 0}\frac{x^4}{\operatorname{ash}^4 x}$$

co když je $+\infty$ (pak $0\cdot\infty$ N.D.F) ??

$y=\operatorname{ash}x$ VoS

$$\overset{\boxed{?}}{=}\frac{1}{4}\,0\cdot\lim_{y\to 0}\frac{\operatorname{sh}^4 y}{y^4}=\frac{1}{4}\cdot 0\cdot 1=\underline{\underline{0}}.$$

↳ NENÍ ∞, tj. OK

Ad: $\frac{\operatorname{sh}y}{y}=\frac{e^{y}-e^{-y}}{2y}=$

$$=\frac{e^{y}-1+1-e^{-y}}{2y}$$

$$=\frac{1}{2}\frac{e^{y}-1}{y}+\frac{1}{2}\frac{e^{-y}-1}{-y}\to\frac{1}{2}+\frac{1}{2}=1,$$

když $y\to 0$.

(?) Zde nejde o neoprávněné část. dosazení, neboť jmenovatel $\neq 0$ a $\neq\infty$, nevede k následnému probl.; lze formulovat a dokázat.