Mathematics for Economists I
Problems 9
Graph of a function

Examine completely the given function, i.e. find its domain, intercepts with axes, limits at the endpoints of D_f, the derivative of the function and its zero points, local and global extrema, intervals of monotony, intervals of convexity/concavity, asymptotes, draw the graph. Justify everything properly.

Remark: problems with an asterisk ($*$) are to be solved without convexity/concavity.

1. $x^3 + 3x^2 - 9x - 11$

2. $x^4 - 4x^3$

3. $\frac{1-2x}{3x^2}$

4. $\frac{3x-1}{1-x}$

5. $\frac{1}{1+e^{-x}}$

6. e^{2-x^2}

7*. $x\sqrt{1-x^2}$

8. $\frac{x^2-x-2}{x-3}$

9*. $\frac{1}{x^2-x-2}$

10. $\log_3(3 + 2x - x^2)$

11. $(3-x)e^x$

12. $x^3 + 2x^2 - 15x$

13. $\sqrt{x^2 + 6x - 16}$

14. $\frac{x^2-5x+4}{x+1}$

15. $\ln(1 - x^2)$

16. $\frac{5+4x-x^2}{x+3}$

17. $\frac{x^2}{2x-8} + 1$

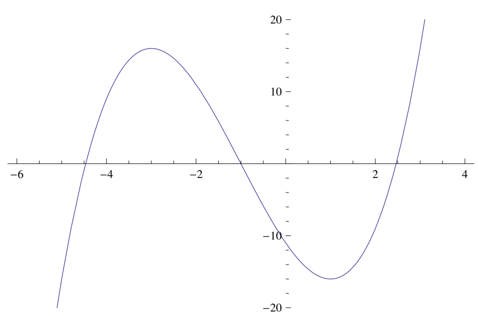
18. $\frac{9-x^2}{2x-10}$

Solutions:

1. $D_f = \mathbb{R}$, roots: $-1, -1 \pm 2\sqrt{3}$, $\lim\limits_{x\to\pm\infty} = \pm\infty$, increases in $(-\infty, -3\rangle$, $\langle 1, +\infty)$, decreases in $\langle -3, 1\rangle$, convex in $\langle -1, +\infty)$, concave in $(-\infty, -1\rangle$, no asymptotes.

2. $D_f = \mathbb{R}$, roots: 0 (triple), 4, $\lim\limits_{x\to\pm\infty} = +\infty$, decreases in $(-\infty, 3\rangle$, increases in $\langle 3, +\infty)$, convex in $(-\infty, 0\rangle$, $\langle 2, +\infty)$, concave in $\langle 0, 2\rangle$, no asymptotes.

1.



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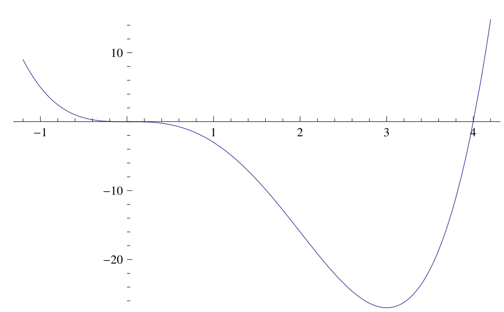
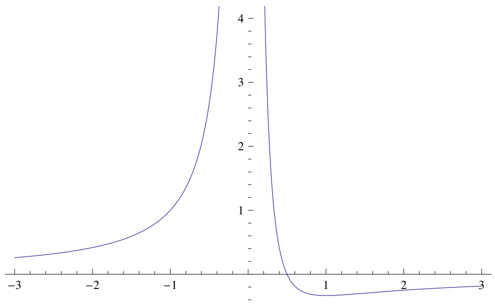
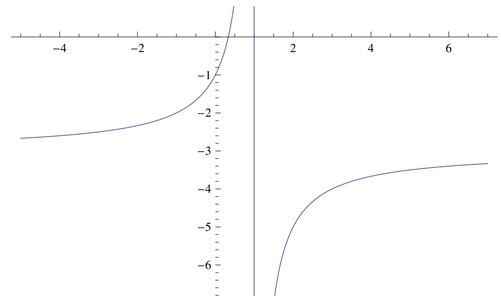


3. $D_f = \mathbb{R}_{-} \cup \mathbb{R}_{+}$, root: $\frac{1}{2}$, $\lim\limits_{x\to 0\pm} = +\infty$, $\lim\limits_{x\to\pm\infty} = 0$, increases in $(-\infty, 0), \langle 1, +\infty)$, decreases in $(0, 1\rangle$, convex in $(-\infty, 0), (0, \frac{3}{2}\rangle$, concave in $\langle \frac{3}{2}, +\infty)$, asymptotes $x = 0$, at $\pm\infty$: $y = 0$.

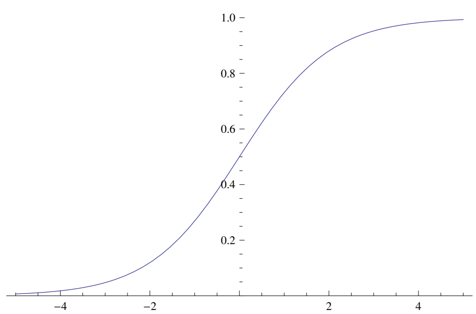
4. $D_f = (-\infty, 1) \cup (1, +\infty)$, root: $\frac{1}{3}$, $\lim\limits_{x\to 1\pm} = \mp\infty$, $\lim\limits_{x\to\pm\infty} = -3$, increases in $(-\infty, 1)$, $(1, +\infty)$, convex in $(-\infty, 1)$, concave in $(1, +\infty)$, asymptotes $x = 1$, at $\pm\infty$: $y = -3$.

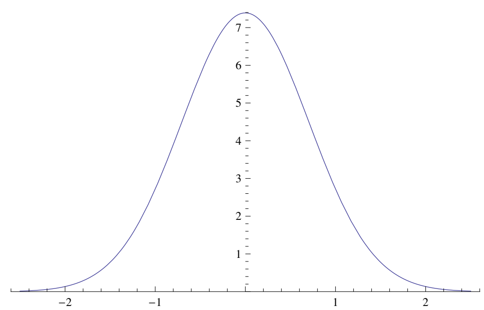
3. 

4. 

5. $D_f = \mathbb{R}$, $f(0) = \frac{1}{2}$, $f(x) > 0$ v $\mathbb{R}$, $\lim\limits_{x\to-\infty} = 0$, $\lim\limits_{x\to+\infty} = 1$, increases in $\mathbb{R}$, convex in $\mathbb{R}_{-}$, concave in $\mathbb{R}_{+}$, asymptote at $-\infty$: $y = 0$, at $+\infty$: $y = 1$.

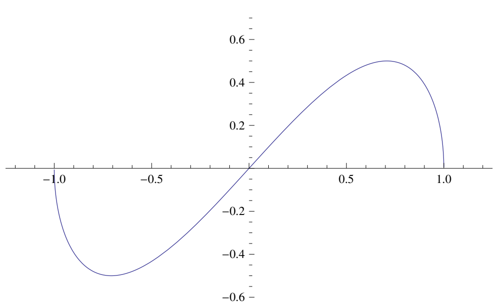
6. $D_f = \mathbb{R}$, $f(0) = e^2$, $f(x) > 0$ v $\mathbb{R}$, $\lim\limits_{x\to\pm\infty} = 0$, increases in $\mathbb{R}_{-}$, decreases in $\mathbb{R}_{+}$, convex in $(-\infty, -\frac{1}{\sqrt{2}}\rangle, \langle \frac{1}{\sqrt{2}}, +\infty)$, concave in $\langle -\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\rangle$, asymptote at $\pm\infty$: $y = 0$.

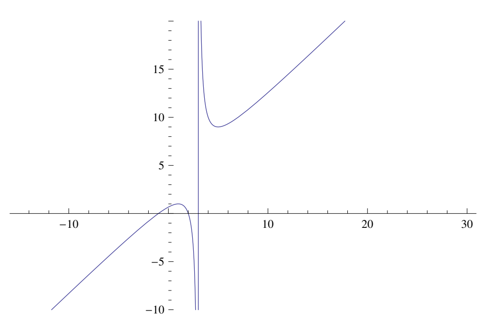
5. 

6. 

7. $D_f = \langle -1, 1\rangle$, $f(-1) = f(0) = f(1) = 0$, decreases in $\langle -1, -\frac{1}{\sqrt{2}}\rangle, \langle \frac{1}{\sqrt{2}}, 1\rangle$, increases in $\langle -\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\rangle$, no asymptotes.

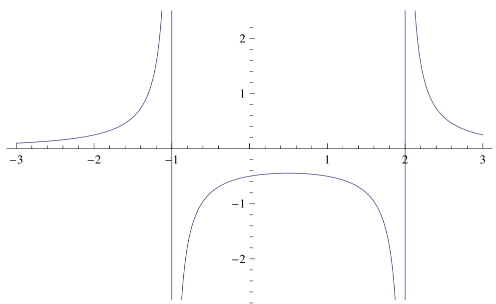
8. $D_f = (-\infty, 3) \cup (3, +\infty)$, $f(0) = \frac{2}{3}$, roots: $-1, 2$, $\lim\limits_{x\to 3\pm} = \pm\infty$, $\lim\limits_{x\to\pm\infty} = \pm\infty$, increases in $(-\infty, 1), (5, +\infty)$, decreases in $(1, 3), (3, 5)$, concave in $(-\infty, 3)$, convex in $(3, +\infty)$, asymptote at $\pm\infty$: $y = x + 2$.

7. 

8. 

9. $D_f = (-\infty, -1) \cup (1, 2) \cup (2, +\infty)$, $f(0) = -\frac{1}{2}$, $f(x) \neq 0$ v $\mathbb{R}$, $\lim\limits_{x\to -1\pm} = \mp\infty$, $\lim\limits_{x\to 2\pm} = \pm\infty$, $\lim\limits_{x\to\pm\infty} = 0$, increases in $(-\infty, -1), (-1, \frac{1}{2})$, decreases in $(\frac{1}{2}, 2), (2, +\infty)$, asymptote at $\pm\infty$: $y = 0$.

10. $D_f = (-1, 3)$, $f(0) = 1$, roots $1 \pm \sqrt{3}$, $\lim\limits_{x\to -1+} = -\infty$, $\lim\limits_{x\to 3-} = -\infty$, increases in $(-1, 1)$, decreases in $(1, 3)$, concave in the whole D_f, asymptotes $x = -1$, $x = 3$.



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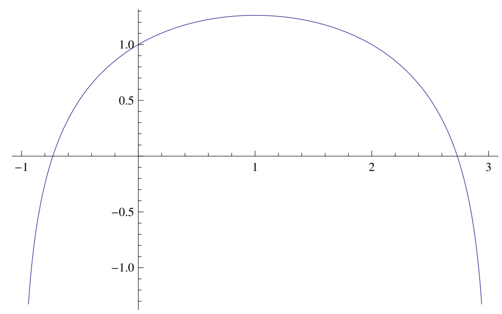
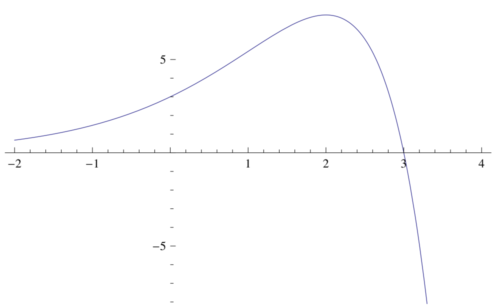


10.

11. $D_f = \mathbb{R}$, root: 3, $\lim\limits_{x\to -\infty} = 0$, $\lim\limits_{x\to +\infty} = -\infty$, increases in $(-\infty, 2\rangle$, decreases in $\langle 2, +\infty\rangle$, convex in $(-\infty, 1\rangle$, concave in $\langle 1, +\infty)$, asymptote $y = 0$ at $-\infty$, no asymptote at $+\infty$.

12. $D_f = \mathbb{R}$, roots: $-5, 0, 3$, $\lim\limits_{x\to\pm\infty} = \pm\infty$, increases in $(-\infty, -3\rangle, \langle\frac{5}{3}, +\infty)$, decreases in $\langle -3, \frac{5}{3}\rangle$, convex in $\langle -\frac{2}{3}, +\infty)$, concave in $(-\infty, -\frac{2}{3}\rangle$, no asymptotes.



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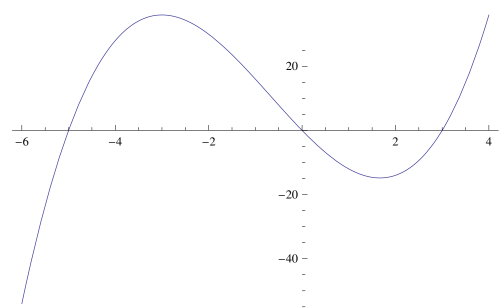

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13. $D_f = (-\infty, -8\rangle \cup \langle 2, +\infty)$, roots: $-8, 2$, $\lim\limits_{x\to\pm\infty} = +\infty$, increases in $\langle 2, +\infty)$, decreases in $\langle -\infty, -8\rangle$, concave in $\langle 2, +\infty)$, concave in $\langle -\infty, -8\rangle$, asymptotes $y = -x - 3$ at $-\infty$, $y = x + 3$ at $+\infty$.

14. $D_f = (-\infty, -1) \cup (-1, +\infty)$, roots: $1, 4$, $\lim\limits_{x\to\pm\infty} = \pm\infty$, $\lim\limits_{x\to -1\pm} = \pm\infty$, increases in $(-\infty, -1-\sqrt{10}\rangle$ and in $\langle -1+\sqrt{10}, +\infty)$, decreases in $\langle -1-\sqrt{10}, -1)$ and in $(-1, -1+\sqrt{10}\rangle$, concave in $(-\infty, -1)$, convex in $(-1, +\infty)$, asymptote $y = x - 6$ at $\pm\infty$.

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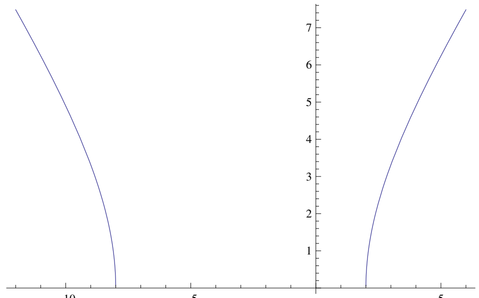

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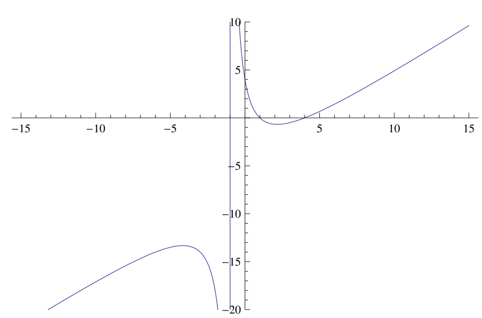
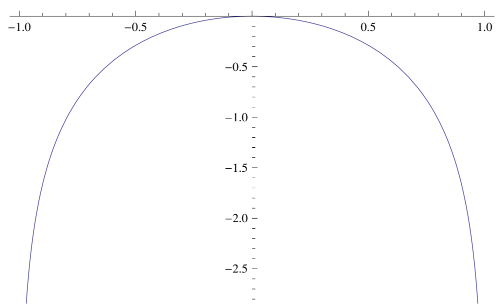


15. $D_f = (-1, 1)$, root: 0, $\lim\limits_{x \to \pm 1} = -\infty$, increases in $(-\infty, 0\rangle$, decreases in $\langle 0, \infty)$, concave in $(-1, 1)$, asymptotes $x = -1,\ x = 1$.

16. locmin $[-7, 18]$, locmax $[1, 2]$, convex in $(-\infty, -3)$, concave in $(-3, +\infty)$, asymptotes $x = -3,\ y = -x + 7$.



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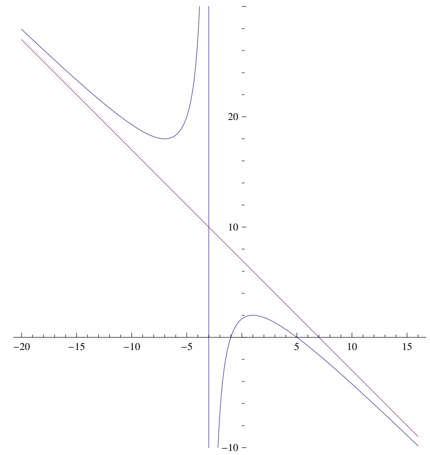

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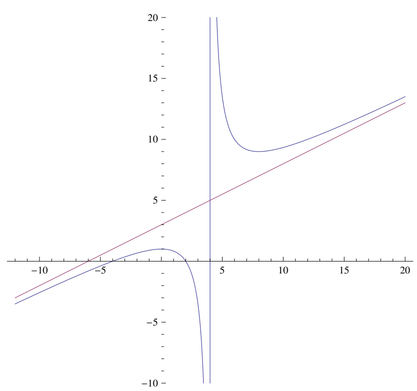
17. locmin $[8, 9]$, locmax $[0, 1]$, concave in $(-\infty, 4)$, convex in $(4, +\infty)$, asymptotes $x = 4,\ y = \frac{x}{2} + 3$.

18. locmin $[1, -1]$, locmax $[9, -9]$, convex in $(-\infty, 5)$, concave in $(5, +\infty)$, asymptotes $x = 5,\ y = \frac{-x-5}{2}$.



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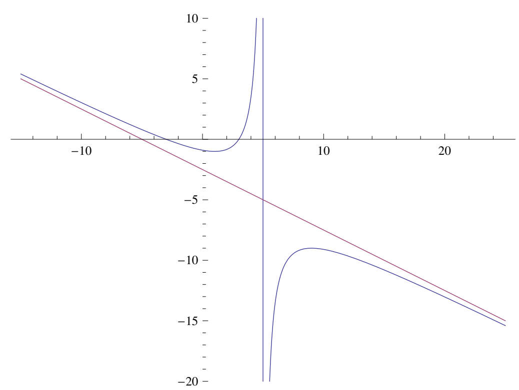

18.