Mathematics for Economists I
Problems 7

Tangent to the graph of a function

1. Consider the function $f(x) = -2x^2 + 3x + 2$. Find the equation of its tangent at the point $x_0 = 1$. Draw the parabola with intercepts with the axes, the vertex, and the specified tangent.

2. A hyperbola is given as a graph of the function

$$f(x) = \frac{2x+1}{x-2}.$$

Determine the equation of the tangent to the graph of f at the point $x_0 = 3$. Draw this hyperbola with intercepts with the axes, center, asymptotes and with the specified tangent. For the tangent, find its intercepts with axes and the contact point with the hyperbola.

3. A parabola is given as a graph of the function $f(x) = 3x^2 + x - 2$. Determine the points $x_0 \in \mathbb{R}$ at which the tangent to the graph of f has the slope equal to -5. At every such point, find the equation of the tangent. Draw the parabola with the intercepts with the axes and its vertex, and the previously found tangent including the contact point with the parabola.

4. A parabola is given as a graph of the function $f(x) = x^2 - 8x + 12$. Determine the points $x_0 \in \mathbb{R}$ at which the tangent to the graph of f has the slope equal to -2. At every such point, find the equation of the tangent. Draw the parabola with the intercepts with the axes and its vertex, and the previously found tangent including the contact point with the parabola.

5. A hyperbola is given as a graph of the function

$$f(x) = \frac{2x+1}{x+3}.$$

Find all points $x_0 \in \mathbb{R}$ at which the tangent line to the hyperbola has its slope equal to 5. At every such point, find the equation of the tangent. Draw the hyperbola including its itercepts with the axes, center and asymptotes. Draw the previously found tangent line(s) into the same picture, including its intercepts with the axes and its contact point(s) with the hyperbola.

Solutions:

1. $y = -x + 4$, contact point $[1, 3]$, roots $-\frac{1}{2}, 2$, vertex $[\frac{3}{4}, \frac{25}{8}]$

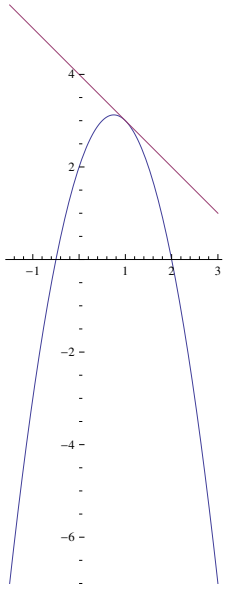
2. $y = -5x + 22$, contact point $[3, 7]$, center $[2, 2]$, intercepts $[0, -1/2], [-1/2, 0]$

3. $x_0 = -1, y = -5x - 5$, contact point $[-1, 0]$, roots $-1, \frac{2}{3}$, vertex $[-\frac{1}{6}, -\frac{25}{12}]$

4. $x_0 = 3, y = -2x + 3$, contact point $[3, -3]$, roots $2, 6$, vertex $[4, -4]$

5. two tangents $y = 5x + 27, y = 5x + 7$; contact points $[-4, 7], [-2, -3]$; center $[-3, 2]$, intercepts $[0, 1/3], [-1/2, 0]$

1.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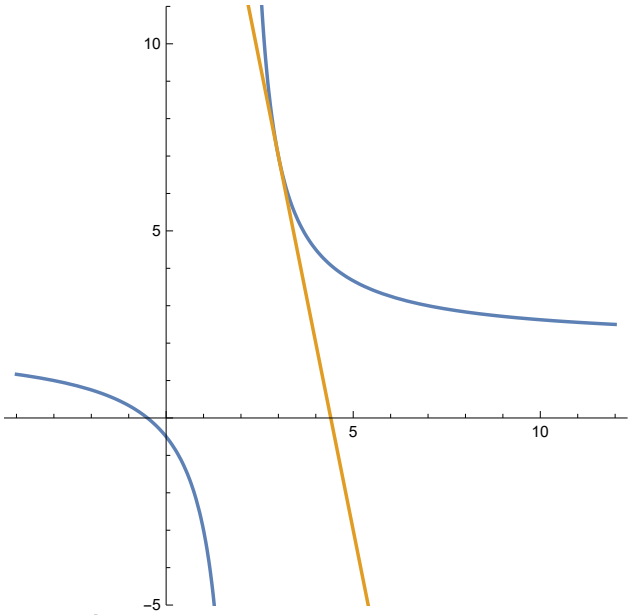
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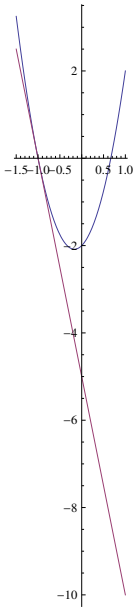
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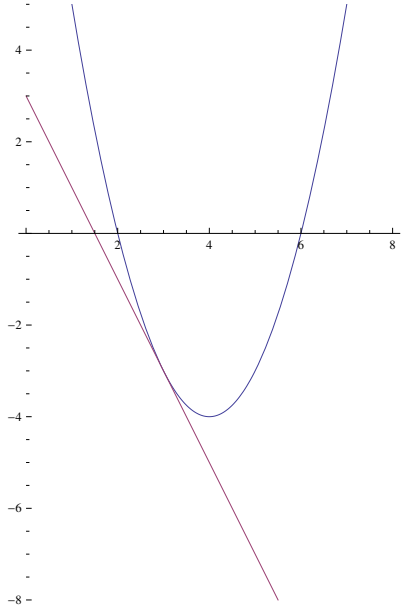
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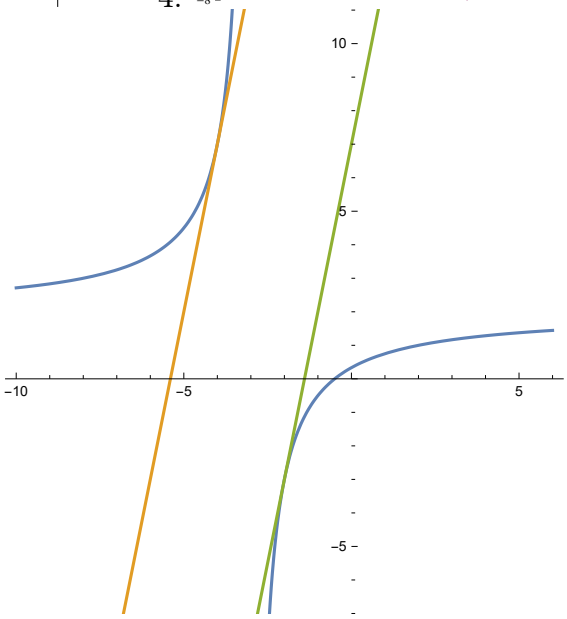
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