

Mathematics for Economists I
Problems 3

Exponential and logarithmic equations

Find all real numbers x satisfying the given equation. Always specify the exact value (maybe as a mathematical expression), and if it is not an integer, write its approximate value rounded to three to four significant figures. You should determine all the solutions without a calculator, only to calculate the numerical value in Problems 3, 17 and 18 you need a calculator.

1. $3^x = 81$

2. $2^x = \frac{1}{64}$

3. $5^x = 17$

4. $10^{x-2} = 1000$

5. $\log_2 x = 10$

6. $3^x + 3^{x+1} + 3^{x+2} + 3^{x+3} = \frac{40}{3}$

7. $3 \cdot (4^x + 9^{x+1}) = 2 \cdot (3 \cdot 4^{x+1} - \frac{9^{x+1}}{4})$

8. $\ln(x+3) - \ln(x-1) = \ln 5$

9. $\ln(x+1) + \ln(x-1) - \ln(x-2) = \ln 8$

10. $\ln(2x-3) + \ln(3x) = \ln(8x-12)$

11. $3 \log_6(2) + \log_6(x+1) = 1 + \log_6(x+2)$

12. $\log_4(x+1) + \log_4(x+6) = 2 \log_4(x+3)$

13. $\sqrt[2x]{2^{3x+1}} = \sqrt[3]{32}$

14. $3^3 \cdot 27^{2x-3} = 81^{3x-5}$

15. $\log_{10}(x+5) - \log_{10}(x-1) = 1 - \log_{10} 5$

16. $\sqrt[2x+4]{4^{x+8}} = \sqrt[4]{64}$

17. A bank offers a fixed interest on deposits of 2% p.a. on the savings account. If we put 10,000 CZK into it, what will be the total amount in the savings account in 5 years?

18. Paul took out a mortgage loan with an interest rate of 4% p.a., let's assume that this interest rate will not change throughout. How many years will he repay if we know that he will eventually pay the bank 1.8 times the borrowed amount?

Solutions:

1. 4. 2. -6. 3. $\log_5 17 = \frac{\ln 17}{\ln 5} \doteq 1,76$. 4. 5. 5. 1024. 6. -1. 7. $-\frac{1}{2}$.
8. 2. 9. 3;5. 10. No solution because the roots $\frac{3}{2}, \frac{4}{3}$ lie out of domain of some of the expressions. 11. 2. 12. 3. 13. 3. 14. $\frac{7}{3}$. 15. 7. 16. 10. 17. 11040,8 CZK. 18. 15 years.