Mathematics for Economists I
Problems 11
Optimization problems – Lagrange multipliers

Find the extremes of the function $f(x, y)$ on the domain $M \subset \mathbb{R}^2$.

1.	$f(x, y) = 6x - 3y$	$M = \{[x, y] \in \mathbb{R}^2;\ x \in \langle -2, 2\rangle,\ x^2 - 4 \leq y \leq 0\}$
2.	$f(x, y) = x - y$	$M = \{[x, y] \in \mathbb{R}^2;\ x \in \langle 0, 2\rangle,\ 0 \leq y \leq e^{x-1}\}$
3.	$f(x, y) = 2x - 2y + 3$	$M = \{[x, y] \in \mathbb{R}^2;\ x^2 + y^2 \leq 2\}$
4.	$f(x, y) = 2x + y - 5$	$M = \{[x, y] \in \mathbb{R}^2;\ x^2 + y^2 \leq 5\}$
5.	$f(x, y) = 3x + 2y$	$M = \{[x, y] \in \mathbb{R}^2;\ x^2 + y^2 \leq 13,\ y \geq 0\}$
6.	$f(x, y) = 7x + y$	$M = \{[x, y] \in \mathbb{R}^2;\ x^2 + y^2 \leq 50,\ y \geq 0, x \geq y\}$
7.	$f(x, y) = x^2 + 10y$	$M = \{[x, y] \in \mathbb{R}^2;\ x^2 + y^2 \leq 169;\ -x - 17 \leq y \leq x + 7\}$
8.	$f(x, y) = xy + x^3$	$M = \{[x, y] \in \mathbb{R}^2;\ -2x - 4 \leq y \leq 2 - x - x^2\}$
9.	$f(x, y) = x^2 + y^2 + 3x - 4y$	$M = \{[x, y] \in \mathbb{R}^2;\ x^2 + y^2 \leq 100;\ x \geq -2y - 10\}$
10.	$f(x, y) = (x - 1)^2 + (y - 2)^2$	$M = \{[x, y] \in \mathbb{R}^2;\ x^2 + y^2 \leq 8;\ 0 \leq y \leq x\}$

Solutions (all candidates):

1. $f(-2, 0) = -12$ MIN, $f(2, 0) = 12$, $f(1, -3) = 15$ MAX

2. $f(0, 0) = 0$, $f(2, 0) = 2$ MAX, $f(0, \frac{1}{e}) = -\frac{1}{e} \doteq -0,37$, $f(2, e) = 2 - e \doteq -0,72$ MIN, $f(1, 1) = 0$

3. $f(-1, 1) = -1$ MIN, $f(1, -1) = 7$ MAX

4. $f(-2, -1) = -10$ MIN, $f(2, 1) = 0$ MAX

5. $f(-\sqrt{13}, 0) = -3\sqrt{13}$ MIN, $f(\sqrt{13}, 0) = 3\sqrt{13}$, $f(3, 2) = 13$ MAX

6. $f(0, 0) = 0$ MIN, $f(7, 1) = 50$ MAX, $f(5, 5) = 40$, $f(\sqrt{50}, 0) = 7\sqrt{50}$

7. $f(-12, -5) = 94$, $f(5, 12) = 145$, $f(-5, -12) = -95$, $f(0, -13) = -130$ MIN, $f(12, 5) = 194$ MAX, $f(-5, 2) = 45$

8. $f(-2; 0) = -8$ MIN, $f(3; -10) = -3$, $f(0; 0) = 0$, $f(2; -8) = -8$ MIN, $f(-2/3; -8/3) = 24/27$, $f(1; 0) = 1$ MAX

9. $f(-10, 0) = 70$, $f(6, -8) = 150$ MAX, $f(-3/2, 2) = -6,25$ MIN, $f(-4, -3) = 25$, $f(-6, 8) = 50$

10. $f(3/2, 3/2) = \frac{1}{2}$ MIN, $f(\sqrt{8}, 0) = (\sqrt{8} - 1)^2 \doteq 3,34$, $f(2, 2) = 1$, $f(-2, -2) = 25$ MAX