Full name: _____

**Sample Midterm test – winter term 2024/25**

In every problem, justify all steps properly.

**1.** (2 points) Find the limit

$$\lim_{n\to\infty} \frac{((n+3)^2-(n-1)^2)\sqrt{9n^2+5n+3}}{(2n+1)^2}$$

**2.** (2 points) Find the limit

$$\lim_{x\to -4} \frac{\ln(x^2+x-11)}{x^2+5x+4}$$

**3.** (2 points) Solve for $x$:

$$\log_2(x+2)+\log_2(3x-2)=2+\log_2(5x+2)$$

**4.** (2 points) Find the derivative of the function

$$f(x)=e^{(3x^2+5x)}+\frac{x-5}{x^2}$$

**5.** (6 points) A parabola is given as a graph of the function

$$f(x)=2x^2-4x-6.$$

- Find the equation of the tangent line to the parabola at the point $x_0=2$.
- Draw the parabola including its intercepts with the axes and vertex.
- Draw the previously found tangent line into the same picture, including its intercepts with the axes and its contact point with the parabola.

**6.** (6 points) Consider the function

$$f(x)=\frac{x^2-x-12}{1-x}$$

- Find its domain $D_f$.
- Find limits at all endpoints of $D_f$.
- Find intercepts of $f$ with axes.
- Find the derivative of $f$.

**Solutions:**

**1.** 6

**2.** $\frac{7}{3}$

**3.** the only solution is 6, while $-\frac{2}{3}$ is a "false root", it does not lie in the domain of $\log_2(3x-2)$

**4.** $e^{3x^2+5x}(6x+5)+\frac{10-x}{x^3}$

**5.** tangent $y=4x-14$, contact point $[2,-6]$, roots $-1,3$, vertex $[1,-8]$

**6.** $D_f=\mathbb{R}-\{1\}$, $\lim\limits_{x\to-\infty} f(x)=\infty$, $\lim\limits_{x\to 1-} f(x)=-\infty$, $\lim\limits_{x\to 1+} f(x)=+\infty$, $\lim\limits_{x\to\infty} f(x)=-\infty$; $P_x:[-3,0],[4,0]$; $P_y=[0,-12]$; $f'(x)=\frac{-x^2+2x-13}{(1-x)^2}$