

Full name: _____

Final test – Winter semester 2025/26
Variant C

In every problem, justify all steps properly.

1. (4 points) Find the limit

$$\lim_{n \rightarrow \infty} \frac{n(\sqrt{n^3 + 2n + 5} - \sqrt{n^3 - 4n + 1})}{\sqrt{9n + 1}}.$$

2. (4 points) Find the derivative of the function

$$f(x) = (4x + 3) \ln(x^2 - x + 4)$$

and find the domain of the function $f(x)$ and of its derivative $f'(x)$.

3. (12 points) A hyperbola is given as a graph of the function

$$f(x) = \frac{2x + 1}{x + 5}.$$

Find all points $x_0 \in \mathbb{R}$ at which the tangent line to the hyperbola has its slope equal to 1. At every such point, find the equation of the tangent. Draw the hyperbola including its intercepts with the axes, center and asymptotes. Draw the previously found tangent line(s) into the same picture, including its intercepts with the axes and its contact point(s) with the hyperbola.

4. (20 points) Examine the behaviour of the function

$$f(x) = \frac{2x^2 - 16x + 14}{x + 1}$$

i.e. find its domain D_f , limits at endpoints of D_f , intercepts with axes, derivative of f , intervals of monotony, global and local extrema, intervals of convexity and concavity, asymptotes. Draw the graph.

5. (20 points) Find the extremes of the function $f(x, y) = x^2 + y^2 - 4y$ on the set

$$M = \{[x, y] \in \mathbb{R}^2; x^2 + y^2 \leq 25; 2x + y + 5 \geq 0\}.$$

For candidates at the curved part of the boundary of M (except for vertices), find the value of λ . Draw the set M including all candidates for an extreme.