Mathematics for Economists I
## Problems 9
Course of the function I

Examine the course of the function, i.e. find its domain, intercepts with axes, limits at the endpoints of D_f, the derivative of the function and its zero points, intervals of monotony, local and global extrema, draw the graph. Justify everything properly.

1. $x^3 + 3x^2 - 9x - 11$ **6.** $e^2 e^{-x^2}$ **11.** $(3 - x)e^x$

2. $x^4 - 4x^3$ **7.** $x\sqrt{1 - x^2}$ **12.** $x^3 + 2x^2 - 15x$

3. $\frac{1-2x}{3x^2}$ **8.** $\frac{x^2-x-2}{x-3}$ **13.** $\sqrt{x^2 + 6x - 16}$

4. $\frac{3x-1}{1-x}$ **9.** $\frac{1}{x^2-x-2}$ **14.** $\frac{x^2-5x+4}{x+1}$

5. $\frac{1}{1+e^{-x}}$ **10.** $\frac{\ln(3+2x-x^2)}{\ln 3}$ **15.** $\ln(1 - x^2)$
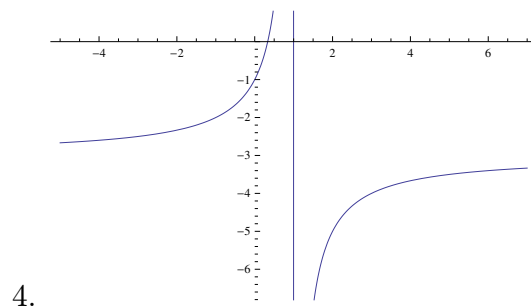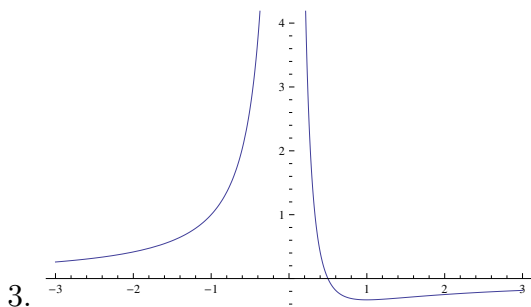
Solutions:

1. $D_f = \mathbb{R}$, roots: $-1, -1 \pm 2\sqrt{3}$, $\lim\limits_{x\to\pm\infty} = \pm\infty$, increases in $(-\infty, -3), \langle 1, +\infty)$, decreases in $\langle -3, 1\rangle$.

2. $D_f = \mathbb{R}$, roots: 0 (triple), 4, $\lim\limits_{x\to\pm\infty} = +\infty$, decreases in $(-\infty, 3)$, increases in $\langle 3, +\infty)$.
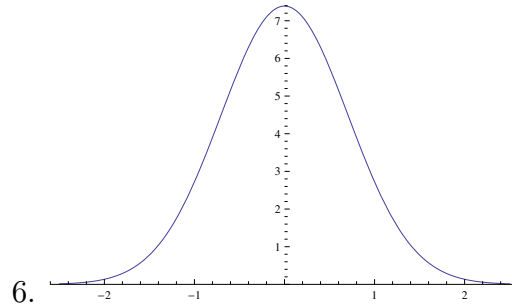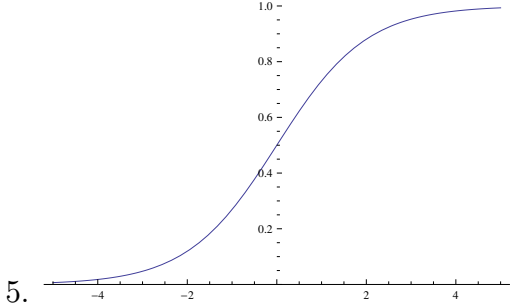

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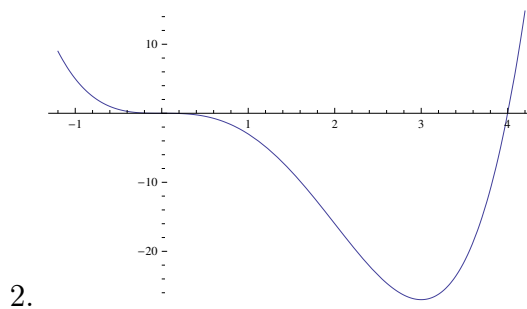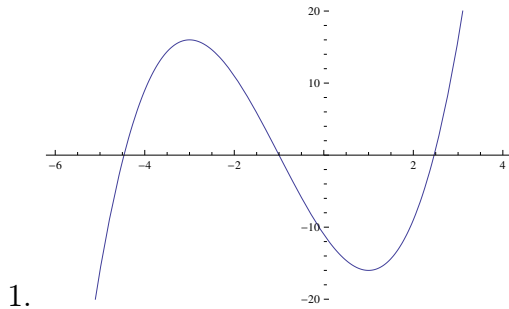
3. $D_f = \mathbb{R}_- \cup \mathbb{R}_+$, root: $\frac{1}{2}$, $\lim\limits_{x\to 0\pm} = +\infty$, $\lim\limits_{x\to\pm\infty} = 0$, increases in $(-\infty, 0), \langle 1, +\infty)$, decreases in $(0, 1\rangle$.

4. $D_f = (-\infty, 1) \cup (1, +\infty)$, root: $\frac{1}{3}$, $\lim\limits_{x\to 1\pm} = \mp\infty$, $\lim\limits_{x\to\pm\infty} = -3$, increases in $(-\infty, 1), (1, +\infty)$.
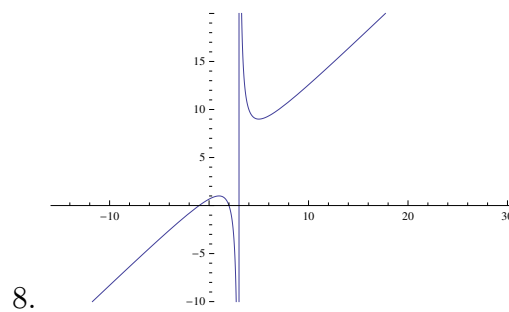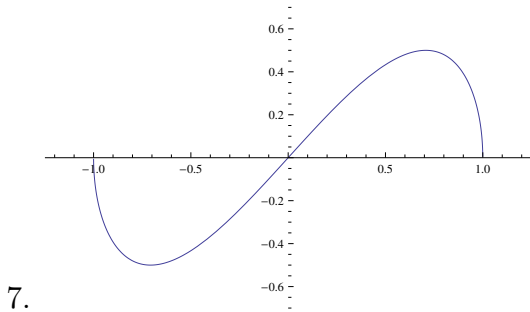

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5. $D_f = \mathbb{R}$, $f(0) = \frac{1}{2}$, $f(x) > 0$ v $\mathbb{R}$, $\lim\limits_{x \to -\infty} = 0$, $\lim\limits_{x \to +\infty} = 1$, increases in $\mathbb{R}$.

6. $D_f = \mathbb{R}$, $f(0) = e^2$, $f(x) > 0$ v $\mathbb{R}$, $\lim\limits_{x \to \pm\infty} = 0$, increases in $\mathbb{R}_-$, decreases in $\mathbb{R}_+$.

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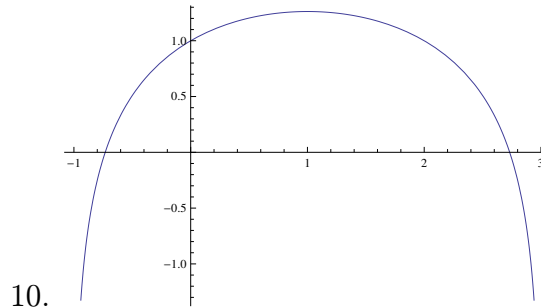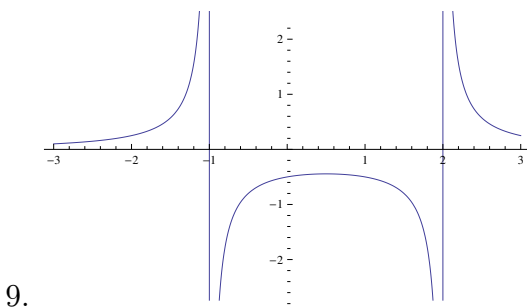**7.** $D_f = \langle -1, 1 \rangle$, $f(-1) = f(0) = f(1) = 0$, decreases in $\langle -1, -\frac{1}{\sqrt{2}} \rangle, \langle \frac{1}{\sqrt{2}}, 1 \rangle$, increases in $\langle -\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \rangle$.

8. $D_f = (-\infty, 3) \cup (3, +\infty)$, $f(0) = \frac{2}{3}$, roots: $-1, 2$, $\lim\limits_{x \to 3\pm} = \pm\infty$, $\lim\limits_{x \to \pm\infty} = \pm\infty$, increases in $(-\infty, 1), (5, +\infty)$, decreases in $(1, 3), (3, 5)$.
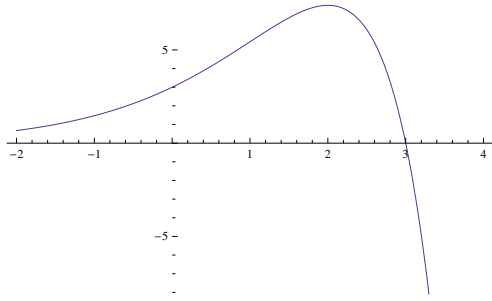
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9. $D_f = (-\infty, -1) \cup (1, 2) \cup (2, +\infty)$, $f(0) = -\frac{1}{2}$, $f(x) \neq 0$ v $\mathbb{R}$, $\lim\limits_{x \to -1\pm} = \mp\infty$, $\lim\limits_{x \to 2\pm} = \pm\infty$, $\lim\limits_{x \to \pm\infty} = 0$, increases in $(-\infty, -1), (-1, \frac{1}{2})$, decreases in $(\frac{1}{2}, 2), (2, +\infty)$.

10. $D_f = (-1, 3)$, $f(0) = 1$, roots $1 \pm \sqrt{3}$, $\lim\limits_{x \to -1+} = -\infty$, $\lim\limits_{x \to 3-} = -\infty$, increases in $(-1, 1)$, decreases in $(1, 3)$.
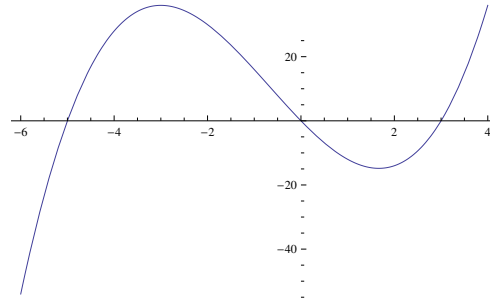
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11. $D_f = \mathbb{R}$, root: 3, $\lim\limits_{x \to -\infty} = 0$, $\lim\limits_{x \to +\infty} = -\infty$, increases in $(-\infty, 2)$, decreases in $\langle 2, +\infty \rangle$.

12. $D_f = \mathbb{R}$, roots: $-5, 0, 3$, $\lim\limits_{x \to \pm\infty} = \pm\infty$, increases in $(-\infty, -3\rangle, \langle\frac{5}{3}, +\infty)$, decreases in $\langle -3, \frac{5}{3}\rangle$.

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13. $D_f = (-\infty, -8\rangle \cup \langle 2, +\infty)$, roots: $-8, 2$, $\lim\limits_{x \to \pm\infty} = +\infty$, increases in $\langle 2, +\infty)$, decreases in $(-\infty, -8\rangle$.

14. $D_f = (-\infty, -1) \cup (-1, +\infty)$, roots: $1, 4$, $\lim\limits_{x \to \pm\infty} = \pm\infty$, $\lim\limits_{x \to -1\pm} = \pm\infty$, increases in $(-\infty, -1 - \sqrt{10}\rangle$ and in $\langle -1 + \sqrt{10}, +\infty)$, decreases in $\langle -1 - \sqrt{10}, -1)$ and in $(-1, -1 + \sqrt{10}\rangle$.

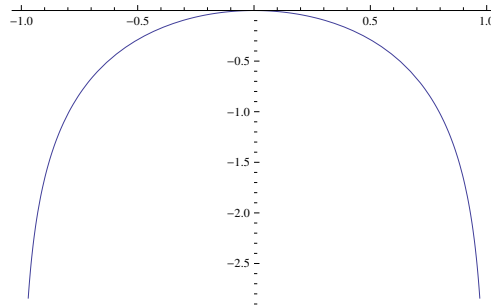**15.** $D_f = (-1, 1)$, root: 0, $\lim\limits_{x \to \pm 1} = -\infty$, increases in $(-\infty, 0\rangle$, decreases in $\langle 0, \infty)$.

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