Mathematics for Economists I
Problems 6
Derivatives

Find the derivative of the given function and the domain where both the function and its derivative are defined.

1. $x^4 + 5x^3 - 2x^2 - 6x + 3$	**2.** $\frac{1}{3x+2}$
3. $\frac{x^2+3x-2}{x+1}$	**4.** $(x^2+1)\ln x$
5. e^{4x-2}	**6.** e^{x^2-x+1}
7. 5^x	**8.** $\ln\sqrt{2x+3}$
9. $\sqrt{x^2-4}$	**10.** $\log_{10}(x^2-1)$
11. $\ln\left(\frac{4-2x}{x+2}\right)$	**12.** $\frac{x+2}{\sqrt{x^2+1}}$
13. $\frac{e^{1+2x}}{x^2+3x+4}$	**14.** $\frac{\ln(3-x)}{x^2+4}$
15. $\sqrt{x^2+\frac{8}{x}}$	**16.** $\frac{\sqrt{2+x}}{4x-2}$
17. $\frac{1}{x}$	**18.** $(x^2+1)^4$

Solutions:

1. $4x^3 + 15x^2 - 4x - 6$, $x \in \mathbb{R}$

2. $\frac{-3}{(3x+2)^2}$, $x \neq -\frac{2}{3}$

3. $\frac{x^2+2x+5}{(x+1)^2}$, $x \neq -1$

4. $2x\ln x + \frac{x^2+1}{x}$, $x \in \mathbb{R}_+$

5. $4e^{4x-2}$, $x \in \mathbb{R}$

6. $(2x-1)e^{x^2-x+1}$, $x \in \mathbb{R}$

7. $(\ln 5)5^x$, $x \in \mathbb{R}$; since $5^x = e^{(\ln 5)x}$, therefore chain rule

8. $\frac{1}{2x+3}$, $x > -\frac{3}{2}$; is possible to differentiate as a superposition of three functions ($\ln z$, $\sqrt{y}$, $2x+3$) or to realize that $\ln\sqrt{y} = \frac{1}{2}\ln y$

9. $\frac{x}{\sqrt{x^2-4}}$, $x \in (-\infty, -2) \cup (2, +\infty)$

10. $\frac{1}{\ln 10}\frac{2x}{x^2-1}$, $x \in (-\infty, -1) \cup (1, +\infty)$; since $\log_{10} y = \frac{\ln y}{\ln 10}$

11. $\frac{4}{x^2-4}$, $x \in (-2, 2)$; is possible to differentiate as a superposition of two functions ($\ln y$, $\frac{4-2x}{x+2}$) or to realize that $\ln\left(\frac{4-2x}{x+2}\right) = \ln(4-2x) - \ln(x+2)$

12. $\frac{1-2x}{(x^2+1)^{\frac{3}{2}}}$, $x \in \mathbb{R}$; we differentiate this as a fraction, and the complicated fraction which results there can be expanded by $\sqrt{x^2+1}$, so we get rid of square roots in the numerator

13. $\frac{e^{1+2x}(2x^2+4x+5)}{(x^2+3x+4)^2}$, $x \in \mathbb{R}$

14. $\frac{-\frac{x^2+4}{3-x}-2x\ln(3-x)}{(x^2+4)^2} = \frac{x^2+4-2x(x-3)\ln(x-3)}{(x^2+4)^2(x-3)}$, $x \in (-\infty, 3)$

15. $\frac{x^3-4}{x^2\sqrt{\frac{x^3+8}{x}}}$, $x \in (-\infty, -2) \cup (0, +\infty)$

16. $\frac{-2x-9}{4(2x-1)^2\sqrt{x+2}}$, $x \in (-2, \frac{1}{2}) \cup (\frac{1}{2}, +\infty)$

17. $\frac{-1}{x^2}$, $x \in \mathbb{R}_- \cup \mathbb{R}_+$; there are two ways: either as a derivative of a fraction or as a derivative of x^{-1}, try both of them

18. $8x(x^2+1)^3$, $x \in \mathbb{R}$; there are two ways: either as a superposition (inner function is x^2+1, outer function is the fourth power) or by expanding the expression and differentiate term by term (which way is faster?)