

**Mathematics for Economists I**  
**Problems 4**

**Limit of a sequence**

Find the limit of the given sequence for  $n \rightarrow \infty$ :

1.  $\frac{n+1}{2n+4}$
2.  $\frac{n^2+n+1}{(n+3)^3+(n-1)^2}$
3.  $\frac{4n^4+(n-3)^2}{n^2-(n+5)^3}$
4.  $\frac{n^2+(n+1)^2}{(n+3)^2+(n+4)^2}$
5.  $\left(\frac{2}{3}\right)^n - \frac{2n-1}{3n+7}$
6.  $\frac{4n-3}{5n^2+6n-1} + (-2)^n$
7.  $\sqrt{n}(\sqrt{n+1} - \sqrt{n})$
8.  $n(\sqrt{n^2+1} - \sqrt{n^2-1})$
9.  $2 \cdot 5^n - 2 \cdot 4^{n+1}$
10.  $\frac{3^{n+1}+5^n}{3^n-5^{n+1}}$
11.  $\frac{3 \cdot 7^{n-1}+2^n}{5 \cdot 3^{n+1}-7^n}$
12.  $\frac{\left(\frac{1}{3}\right)^n + \left(\frac{1}{2}\right)^n}{\left(\frac{1}{4}\right)^n - \left(\frac{1}{2}\right)^n}$
13.  $\sqrt{n^2+n+1} - \sqrt{n^2-n-1}$
14.  $\frac{\sqrt{n+4} \cdot \sqrt{4n+9} \cdot \sqrt{9n+16}}{(n+1)^2 - (n-1)^2}$
15.  $\frac{\left(\frac{4}{3}\right)^{n+1} + \left(\frac{6}{5}\right)^{2n+1}}{\left(\frac{7}{5}\right)^{n+2} - \left(\frac{6}{5}\right)^{2n}}$
16.  $\frac{\sqrt{n^3+3n^2} - \sqrt{n^3-3n^2}}{\sqrt{n}}$
17.  $\frac{\sqrt[3]{n+2} \cdot \sqrt[3]{n^2+2n+4}}{(n+1)^2 - (n-1)^2}$
18.  $\frac{(n+2)^3 - (n-2)^3}{(2n+1)^2}$
19.  $\frac{3 \cdot \left(\frac{1}{3}\right)^n + 5 \cdot \left(\frac{2}{5}\right)^n}{\left(\frac{1}{3}\right)^n + \left(\frac{2}{5}\right)^n}$

20. You sell your used bicycle for CZK 10 000; you want to make it cheaper gradually. Two options came to your mind: either to reduce the price by CZK 1 000 every month or to reduce the price by 20% every month. Analyze and compare them clearly. What would happen if time went „to infinity“?

**Solutions:** 1.  $\frac{1}{2}$ , 2. 0, 3.  $-\infty$ , 4. 1, 5.  $-\frac{2}{3}$ , 6. does not exist,  
7.  $\frac{1}{2}$ , 8. 1, 9.  $+\infty$ , 10.  $-\frac{1}{5}$ , 11.  $-\frac{3}{7}$ , 12.  $-1$ , 13. 1, 14.  $+\infty$ ,  
15.  $-\frac{6}{5}$ , 16. 3, 17.  $\frac{1}{4}$ , 18. 3, 19. 5, 20. left upon your own work.