Mathematics for Economists I
Problems 11

Partial derivatives

Find partial derivatives of given functions with respect to all variables.

1. $f(x, y) = x^3 y^2 + xy^3$

2. $f(x, y) = e^{xy^2}$

3. $f(x, y, z) = \ln(xz) + \frac{y}{z} - xy$

Stationary points of functions of several variables

Find all stationary points of the given function (consider $x, y, z \in \mathbb{R}$ if not mentioned otherwise).

4. $x^3 - 3x^2 + 4xy + y^2 + 8x$

5. $x^2 - 2xy + y^3 + 6y^2 + 3y$

6. $y^4 + 32x^2 - 32xy$

7. $x - y^2 - e^{x-2y}$

8. $x^3 + y^3 - 9xy + 15$

9. $\ln(x + 1) - xy^2; \quad x \in (-1, \infty), y \in \mathbb{R}$

10. $e^{x^2 + (y+2)^2} + x^2$

11. $x^2 - 6x + y^2 + 2y + z^2 - 4z$

12. $xy - 2xz + 3yz + 7x - 15y + 3z$

Stationary points and extremes on polygons (substitution method)

For every given function f:
a) find all its stationary points in $\mathbb{R}^2$;
b) find its extremes on the triangle A, B, C.

13. $f(x, y) = x^2 - 6xy - 3y^2 - 8x$, $A = [0, 1], B = [0, -2], C = [3, -2]$.

14. $f(x, y) = -3x^2 + 6xy + y^2 - 6x + 6y$, $A = [-2, 1], B = [1, 1], C = [-2, -2]$.

15. $f(x, y) = 4x^2 - 2xy + y^2 - 6y$, $A = [0, 2], B = [4, 2], C = [0, 6]$.

16. $f(x, y) = x^2 + 2xy + 4y^2 + 2x + 8y$, $A = [-2, 0], B = [2, 0], C = [-2, -4]$

17. $f(x, y) = x^2 + 4xy + 2y^2 + 2x + 4y$, $A = [1, 2], B = [1, -2], C = [-1, -2]$.

Solutions:

1. $\partial_x f(x,y) = 3x^2 y^2 + y^3$
$\partial_y f(x,y) = 2x^3 y + 3xy^2$

2. $\partial_x f(x,y) = y^2 e^{xy^2}$
$\partial_y f(x,y) = 2xy e^{xy^2}$

3. $\partial_x f(x,y,z) = \frac{1}{x} - y$
$\partial_y f(x,y,z) = \frac{1}{z} - x$
$\partial_z f(x,y,z) = \frac{1}{z} + \frac{-y}{z^2}$ (all for $xz > 0$)

4. $[2/3, -4/3], [4, -8],$

5. $[-3, -3], [-1/3, -1/3],$

6. $[0, 0], [1, 2], [-1, -2],$

7. $[2, 1],$

8. $[0, 0], [3, 3],$

9. $[0, 1], [0, -1],$

10. $[0, -2],$

11. $[3, -1, 2],$

12. $[3, 1, 4].$

	stat. point	min	max
13.	$(1, -1)$	$f(0, -2) = -12$	$f(3, -2) = 9$
14.	$(-1, 0)$	$f(-2, 1) = -5$	$f(-2, -2) = 16$
15.	$(1, 4)$	$f(1, 4) = -12$	$f(4, 2) = 40$
16.	$(0, -1)$	$f(0, -1) = -4$	$f(-2, -4) = 48$
17.	$(-1, 0)$	$f(1, -2) = -5$	$f(1, 2) = 27$