Mathematics for Economists I
Problems 10
Course of the function II (convexity/concavity, asymptotes)

Examine the course of the function, i.e. find its domain, intercepts with axes, limits at the endpoints of D_f, the derivative of the function and its zero points, local and global extrema, intervals of monotony, intervals of convexity/concavity, asymptotes, draw the graph. Justify everything properly.

Problems 1.–15. are the same as in the sheet "Problems 9", just the questions of **asymptotes** and **convexity/concavity** are added (unless stated otherwise: solve problems with an asterisk ($*$) without convexity/concavity).

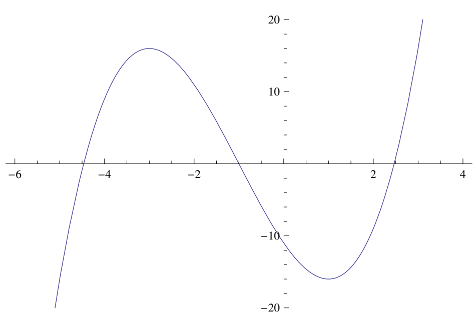
1. $x^3+3x^2-9x-11$	**6.** $e^2e^{-x^2}$	**11.** $(3-x)e^x$
2. x^4-4x^3	**7*.** $x\sqrt{1-x^2}$	**12.** x^3+2x^2-15x
3. $\frac{1-2x}{3x^2}$	**8.** $\frac{x^2-x-2}{x-3}$	**13.** $\sqrt{x^2+6x-16}$
4. $\frac{3x-1}{1-x}$	**9*.** $\frac{1}{x^2-x-2}$	**14.** $\frac{x^2-5x+4}{x+1}$
5. $\frac{1}{1+e^{-x}}$	**10.** $\frac{\ln(3+2x-x^2)}{\ln 3}$	**15.** $\ln(1-x^2)$

16. $\frac{5+4x-x^2}{x+3}$ **17.** $\frac{x^2}{2x-8}+1$ **18.** $\frac{9-x^2}{2x-10}$

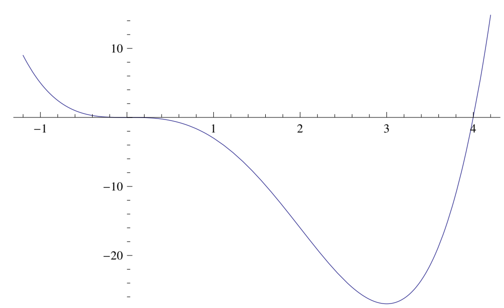
Solutions:

1. $D_f=\mathbb{R}$, roots: $-1, -1\pm 2\sqrt{3}$, $\lim_{x\to\pm\infty}=\pm\infty$, increases in $(-\infty,-3\rangle, \langle 1,+\infty)$, decreases in $\langle -3,1\rangle$, convex in $\langle -1,+\infty)$, concave in $(-\infty,-1\rangle$, no asymptotes.

2. $D_f=\mathbb{R}$, roots: 0 (triple), 4, $\lim_{x\to\pm\infty}=+\infty$, decreases in $(-\infty,3\rangle$, increases in $\langle 3,+\infty)$, convex in $(-\infty,0\rangle, \langle 2,+\infty)$, concave in $\langle 0,2\rangle$, no asymptotes.



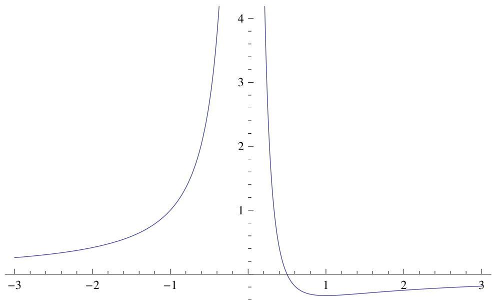
1.

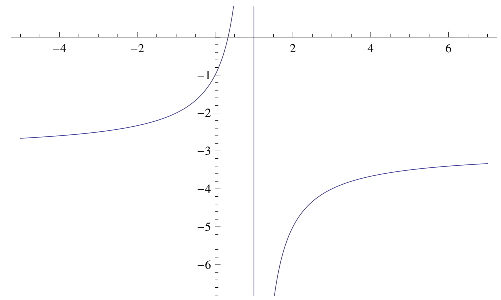


2.

3. $D_f = \mathbb{R}_- \cup \mathbb{R}_+$, root: $\frac{1}{2}$, $\lim\limits_{x\to 0\pm} = +\infty$, $\lim\limits_{x\to\pm\infty} = 0$, increases in $(-\infty, 0), \langle 1, +\infty)$, decreases in $(0, 1\rangle$, convex in $(-\infty, 0), (0, \frac{3}{2}\rangle$, concave in $\langle \frac{3}{2}, +\infty)$, asymptotes $x = 0$, at $\pm\infty$: $y = 0$.

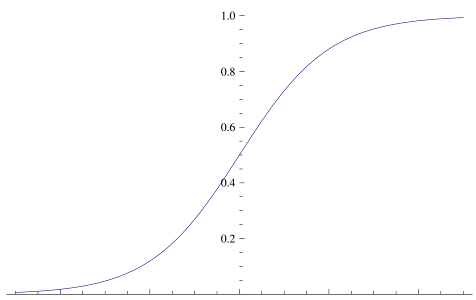
4. $D_f = (-\infty, 1) \cup (1, +\infty)$, root: $\frac{1}{3}$, $\lim\limits_{x\to 1\pm} = \mp\infty$, $\lim\limits_{x\to\pm\infty} = -3$, increases in $(-\infty, 1)$, $(1, +\infty)$, convex in $(-\infty, 1)$, concave in $(1, +\infty)$, asymptotes $x = 1$, at $\pm\infty$: $y = -3$.

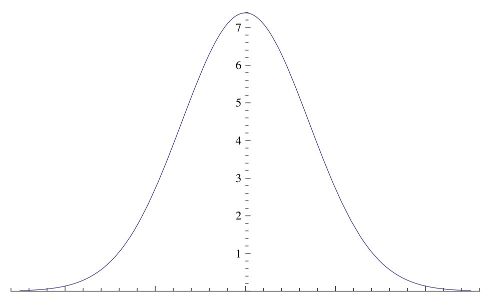
3. 

4. 

5. $D_f = \mathbb{R}$, $f(0) = \frac{1}{2}$, $f(x) > 0$ v $\mathbb{R}$, $\lim\limits_{x\to-\infty} = 0$, $\lim\limits_{x\to+\infty} = 1$, increases in $\mathbb{R}$, convex in $\mathbb{R}_-$, concave in $\mathbb{R}_+$, asymptote at $-\infty$: $y = 0$, at $+\infty$: $y = 1$.

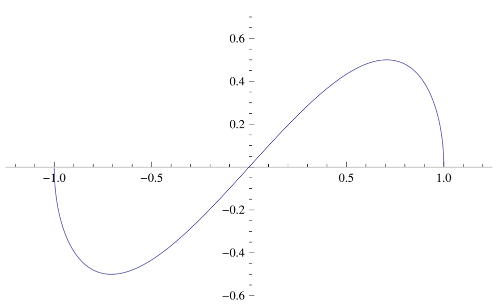
6. $D_f = \mathbb{R}$, $f(0) = e^2$, $f(x) > 0$ v $\mathbb{R}$, $\lim\limits_{x\to\pm\infty} = 0$, increases in $\mathbb{R}_-$, decreases in $\mathbb{R}_+$, convex in $(-\infty, -\frac{1}{\sqrt{2}}\rangle, \langle \frac{1}{\sqrt{2}}, +\infty)$, concave in $\langle -\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\rangle$, asymptote at $\pm\infty$: $y = 0$.

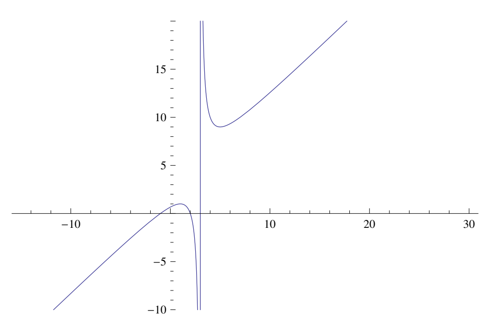
5. 

6. 

7. $D_f = \langle -1, 1\rangle$, $f(-1) = f(0) = f(1) = 0$, decreases in $\langle -1, -\frac{1}{\sqrt{2}}\rangle, \langle \frac{1}{\sqrt{2}}, 1\rangle$, increases in $\langle -\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\rangle$, no asymptotes.

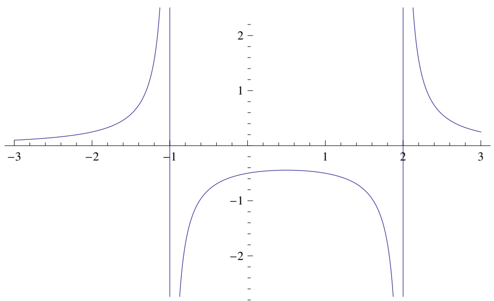
8. $D_f = (-\infty, 3) \cup (3, +\infty)$, $f(0) = \frac{2}{3}$, roots: $-1, 2$, $\lim\limits_{x\to 3\pm} = \pm\infty$, $\lim\limits_{x\to\pm\infty} = \pm\infty$, increases in $(-\infty, 1\rangle, \langle 5, +\infty)$, decreases in $\langle 1, 3), (3, 5\rangle$, concave in $(-\infty, 3)$, convex in $(3, +\infty)$, asymptote at $\pm\infty$: $y = x + 2$.

7. 

8. 

9. $D_f = (-\infty,-1) \cup (1,2) \cup (2,+\infty)$, $f(0) = -\frac{1}{2}$, $f(x) \neq 0$ v $\mathbb{R}$, $\lim\limits_{x\to-1\pm} = \mp\infty$, $\lim\limits_{x\to 2\pm} = \pm\infty$, $\lim\limits_{x\to\pm\infty} = 0$, increases in $(-\infty,-1), (-1,\frac{1}{2})$, decreases in $(\frac{1}{2},2), (2,+\infty)$, asymptote at $\pm\infty$: $y = 0$.

10. $D_f = (-1,3)$, $f(0) = 1$, roots $1 \pm \sqrt{3}$, $\lim\limits_{x\to-1+} = -\infty$, $\lim\limits_{x\to 3-} = -\infty$, increases in $(-1,1)$, decreases in $(1,3)$, concave in the whole D_f, asymptotes $x = -1$, $x = 3$.



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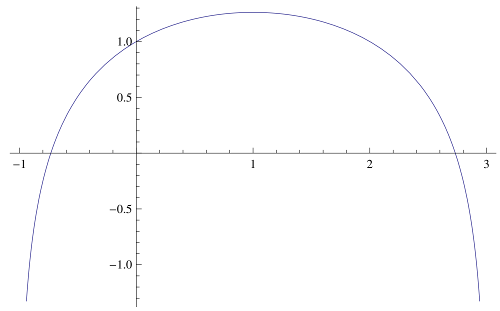
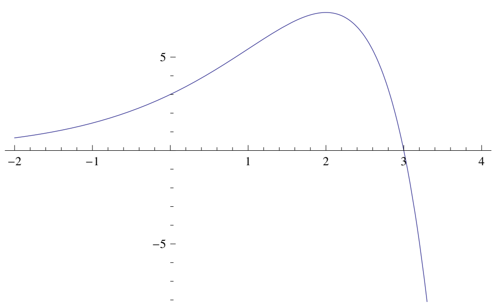


10.

11. $D_f = \mathbb{R}$, root: 3, $\lim\limits_{x\to-\infty} = 0$, $\lim\limits_{x\to+\infty} = -\infty$, increases in $(-\infty,2\rangle$, decreases in $\langle 2,+\infty)$, convex in $(-\infty,1\rangle$, concave in $\langle 1,+\infty)$, asymptote $y = 0$ at $-\infty$, no asymptote at $+\infty$.

12. $D_f = \mathbb{R}$, roots: $-5, 0, 3$, $\lim\limits_{x\to\pm\infty} = \pm\infty$, increases in $(-\infty,-3\rangle, \langle\frac{5}{3},+\infty)$, decreases in $\langle -3,\frac{5}{3}\rangle$, convex in $\langle -\frac{2}{3},+\infty)$, concave in $(-\infty,-\frac{2}{3}\rangle$, no asymptotes.



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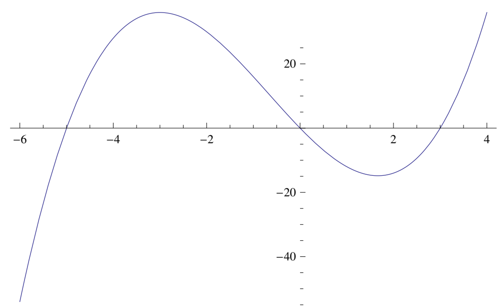
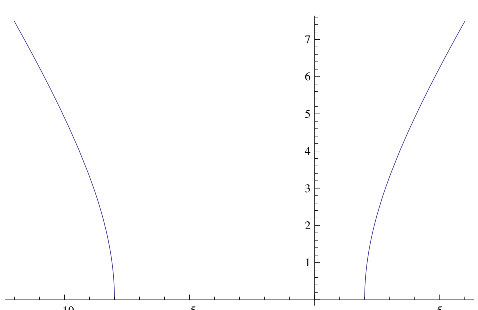


12.

13. $D_f = (-\infty,-8\rangle \cup \langle 2,+\infty)$, roots: $-8, 2$, $\lim\limits_{x\to\pm\infty} = +\infty$, increases in $\langle 2,+\infty)$, decreases in $\langle -\infty,-8\rangle$, concave in $\langle 2,+\infty)$, concave in $\langle -\infty,-8\rangle$, asymptotes $y = -x - 3$ at $-\infty$, $y = x + 3$ at $+\infty$.

14. $D_f = (-\infty,-1) \cup (-1,+\infty)$, roots: $1, 4$, $\lim\limits_{x\to\pm\infty} = \pm\infty$, $\lim\limits_{x\to-1\pm} = \pm\infty$, increases in $(-\infty,-1-\sqrt{10}\rangle$ and in $\langle -1+\sqrt{10},+\infty)$, decreases in $\langle -1-\sqrt{10},-1)$ and in $(-1,-1+\sqrt{10}\rangle$, concave in $(-\infty,-1)$, convex in $(-1,+\infty)$, asymptote $y = x - 6$ at $\pm\infty$.



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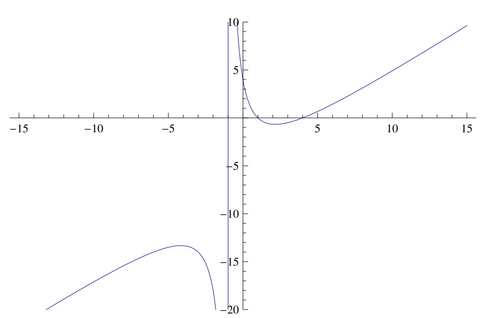

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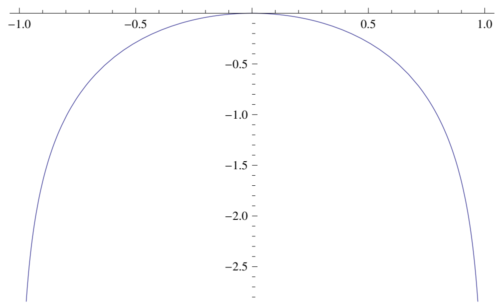
15. $D_f = (-1,1)$, root: 0, $\lim\limits_{x\to\pm 1} = -\infty$, increases in $(-\infty, 0\rangle$, decreases in $\langle 0,\infty)$, concave in $(-1,1)$, asymptotes $x=-1,\ x=1$.

16. locmin $[-7,18]$, locmax $[1,2]$, convex in $(-\infty,-3)$, concave in $(-3,+\infty)$, asymptotes $x=-3,\ y=-x+7$.



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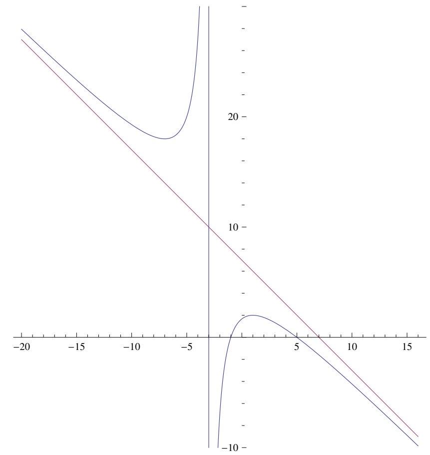

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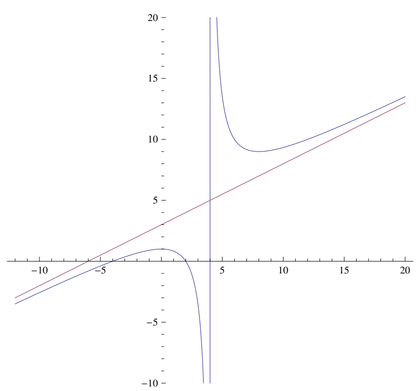
17. locmin $[8,9]$, locmax $[0,1]$, concave in $(-\infty,4)$, convex in $(4,+\infty)$, asymptotes $x=4,\ y=\frac{x}{2}+3$.

18. locmin $[1,-1]$, locmax $[9,-9]$, convex in $(-\infty,5)$, concave in $(5,+\infty)$, asymptotes $x=5,\ y=\frac{-x-5}{2}$.



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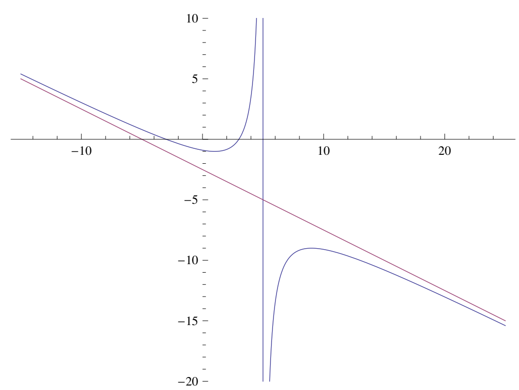

18.