

Full name: _____

Midterm test – winter term 2023/24
Variant A

In every problem, justify all steps properly.

1. (3 points) Find the limit

$$\lim_{n \rightarrow \infty} (\sqrt{4n+2}) \cdot (\sqrt{n^3+3n+7} - \sqrt{n^3-n+1}).$$

2. (3 points) Find the derivative of the function

$$f(x) = e^{2x^2+x} + \frac{x+1}{2-x}$$

and find the domain of definition of the function $f(x)$ and of its derivative $f'(x)$.

3. (6 points) A parabola is given as a graph of the function

$$f(x) = x^2 - 8x + 12.$$

Find the equation of the tangent line to the parabola at the point $x_0 = 3$. Draw the parabola including its intercepts with the axes and vertex. Draw the previously found tangent line into the same picture, including its intercepts with the axes and its contact point with the parabola.

4. (10 points) Examine the course of the function

$$f(x) = (2-x)e^x,$$

i.e. find its domain of definition D_f , limits at endpoints of D_f , intercepts with axes, derivative of f , intervals of monotony, global and local extrema. Draw the graph.

Hint: $e \doteq 2,71$, $e^2 \doteq 7,39$, $\frac{2}{e} \doteq 0,74$

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Variant B

In every problem, justify all steps properly.

1. (3 points) Find the limit

$$\lim_{n \rightarrow \infty} \frac{2^{2n+1} + 5 \cdot 3^{n+1}}{\left(\frac{7}{2}\right)^n - 4^{n-1}}.$$

2. (3 points) Find the derivative of the function

$$f(x) = \ln(2x^2 + 2x + 5) + \frac{x - 3}{x + 1}$$

and find the domain of definition of the function $f(x)$ and of its derivative $f'(x)$.

3. (6 points) A hyperbola is given as a graph of the function

$$f(x) = \frac{2x - 1}{x + 3}.$$

Find the equation of the tangent line to the hyperbola at the point $x_0 = -2$. Draw the hyperbola including its intercepts with the axes, center and asymptotes. Draw the previously found tangent line into the same picture, including its intercepts with the axes and its contact point with the hyperbola.

4. (10 points) Examine the course of the function

$$f(x) = x^3 + 5x^2 - 8x - 12,$$

i.e. find its domain of definition D_f , limits at endpoints of D_f , intercepts with axes, derivative of f , intervals of monotony, global and local extrema. Draw the graph.

Hint: $\frac{10}{27} \doteq 0,37$, $\frac{286}{27} \doteq 10,59$, $\frac{400}{27} \doteq 14,81$.