Matematická analýza NMTM101, ZS 22/23, cvičení s L. Krumpem

Cvičení č. 4: Limity posloupností II

1. Vypočtěte limity:

(a) $\lim\limits_{n\to\infty} \frac{\sqrt{4n^2+3n+5}}{(n+2)^2-(n-1)^2}$

(b) $\lim\limits_{n\to\infty} \frac{\sqrt[3]{n^2}}{n+1}$

(c) $\lim\limits_{n\to\infty} \frac{3\cdot 7^{n-1}+2^n}{5\cdot 3^{n+1}-7^n}$

(d) $\lim\limits_{n\to\infty} \frac{\left(\frac{7}{5}\right)^n-\left(\frac{6}{5}\right)^{2n+1}}{\left(\frac{5}{4}\right)^n+\left(\frac{144}{100}\right)^n}$

(e) $\lim\limits_{n\to\infty} \sqrt{n^2+n+1}-\sqrt{n^2-n-1}$

(f) $\lim\limits_{n\to\infty} \frac{\sqrt{n^3+3n^2}-\sqrt{n^3-3n^2}}{\sqrt{n}}$

(g) $\lim\limits_{n\to\infty} \sqrt{n}(\sqrt{n^3+3n+1}-\sqrt{n^3+3n-1})$

(h) $\lim\limits_{n\to\infty} \frac{\sqrt[3]{27n-1}\cdot\sqrt[3]{(n+1)^2}}{(2n-3)(2n+3)-16(\frac{1}{2}n+1)^2}$

(i) $\lim\limits_{n\to\infty} \frac{5\cdot\left(\frac{2}{5}\right)^{4n}-3\cdot\left(\frac{4}{5}\right)^{3n}+2\cdot(0{,}5)^n}{8\cdot\left(\frac{25}{100}\right)^{\frac{n}{2}}+4\cdot(0{,}8)^{3n}+\frac{1}{7^{2n}}}$

(j) $\lim\limits_{n\to\infty} \frac{n^2\sqrt{(4n-3)^3-16n(2n-4)^2}}{n((n\cdot\sqrt{3}-2)^2+(2n+3)^2)}$

(k) $\lim\limits_{n\to\infty} \frac{\sqrt{n+\sqrt{n+\sqrt{n}}}}{\sqrt{n+1}}$

(l) $\lim\limits_{n\to\infty} n^{\frac{4}{3}}(\sqrt[3]{n^2+1}-\sqrt[3]{n^2-1})$

Pomůcka (porovnání logaritmu, mocniny, exponenciály a faktoriálu):

$\lim_{n\to\infty} \frac{\log_a n}{n^k} = 0$ pro $a > 0$ $(a \neq 1), k \in \mathbb{R}_+$

$\lim_{n\to\infty} \frac{n^k}{a^n} = 0$ pro $a > 1, k \in \mathbb{R}$

$\lim_{n\to\infty} \frac{a^n}{n!} = 0$ pro $a \in \mathbb{R}$

$\lim_{n\to\infty} \sqrt[n]{a} = 1$ pro $a > 0$

$\lim_{n\to\infty} \sqrt[n]{n} = 1$

$\lim_{n\to\infty} \sqrt[n]{n!} = \infty$

$\lim_{n\to\infty} (1 + \frac{1}{n})^n = e, \ \lim_{n\to\infty} (1 + \frac{x}{n})^n = e^x$ pro $x \in \mathbb{R}$

2. Vypočtěte limity:

(a) $\lim_{n\to\infty} \frac{n^3+4n^2-5n}{2^n}$

(b) $\lim_{n\to\infty} \frac{\ln(n+2)+\ln(n+3)}{n^2}$

(c) $\lim_{n\to\infty} \frac{\log_5 n+n^3+2n^2+2e^n+4^n}{n^4-e^n+\ln(2n)-4^n+3}$

(d) $\lim_{n\to\infty} \frac{(n+1)^8}{n!}$

(e) $\lim_{n\to\infty} \frac{\ln(n^2+\sqrt{n})}{\ln(\sqrt[3]{n})}$

(f) $\lim_{n\to\infty} (1 + \frac{1}{2n})^{8n+7}$

(g) $\lim_{n\to\infty} \frac{\ln(1+\sqrt{n}+\sqrt[3]{n})}{\ln(1+\sqrt[3]{n}+\sqrt[4]{n})}$

(h) $\lim_{n\to\infty} \frac{\sin(n^2)}{n}$

Řešení:

1. (a) $\frac{2}{5}$; (b) 0; (c) $-\frac{3}{7}$; (d) $-\frac{6}{5}$; (e) 1; (f) 3; (g) 0; (h) $-\frac{3}{16}$; (i) $-\frac{3}{4}$; (j) $\frac{4\sqrt{7}}{7}$; (k) 1; (l) $\frac{2}{3}$.

2. (a) 0; (b) 0; (c) -1; (d) 0; (e) 6; (f) e^4; (g) $\frac{3}{2}$; (h) 0.