Fixed interval scheduling problems under uncertainty

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Stochastic Programming and Approximation, 2016

Contents

Contents

Motivation

Prof. Asmund Olstad (Molde University, Norway) – reparations of oil platforms:

Fixed Interval Scheduling (FIS) problem under Uncertainty (FISuU)

- **Machines** = highly specialized and costly workmen (Ph.D. in engineering, mountain climbing etc.)
- **Jobs** = reparations of oil platforms
- **Intervals** = helicopter trips
- **Uncertainty** = weather conditions, unpredictable complications

Fixed Interval Scheduling

Other applications:

- personnel scheduling (Shift Minimization Personnel Task Scheduling Problems)
- assigning aircrafts to gates (Kroon et al. 1995)
- bus driver scheduling problem
- crew scheduling,
- vehicle scheduling
- telecommunication, data transmission
- scheduling of operating rooms in hospitals
- ...

Contents

Notation

- $\mathcal{T} = [0, T]$ – planning horizon with continuous time,
- $\mathcal{C}$ – set of machines,
- $\mathcal{J}$ – set of jobs,
    - s_j – starting time,
    - f_j^0 – prescribed completion time,
    - $D_j(\xi)$ – random delay, where $D_j(\xi) = 0$ has a positive probability,
    - random "true" finishing time

$$f_j(\xi) = f_j^0 + D_j(\xi),$$

- x_{jc} – binary decision variable, $j \in \mathcal{J}$, $c \in \mathcal{C}$ – equal to one if job j is assigned to machine c, and to zero otherwise.

Maximization of schedule reliability

$$\max_{x_{jc}} P\left(\xi \in \Xi : \sum_{j:\ s_j \le t < f_j(\xi)} x_{jc} \le 1,\ t \in \hat{\mathcal{T}}, c \in \mathcal{C}\right)$$

$$\begin{aligned} \sum_{j:\ s_j \le t < f_j^0} x_{jc} &\le 1,\ c \in \mathcal{C},\ t \in \hat{\mathcal{T}}, \\ \sum_{c \in \mathcal{C}} x_{jc} &= 1,\ j \in \mathcal{J},\ x_{jc} \in \{0,1\},\ c \in \mathcal{C}, j \in \mathcal{J}. \end{aligned} \tag{1}$$

- Maximization of probability that in each moment a machine processes at most one job under the constraints:
- at most one job assigned to a machine at each time with respect to the prescribed job processing times,
- a job is assigned to exactly one machine.

Minimization of expected number of overlaps

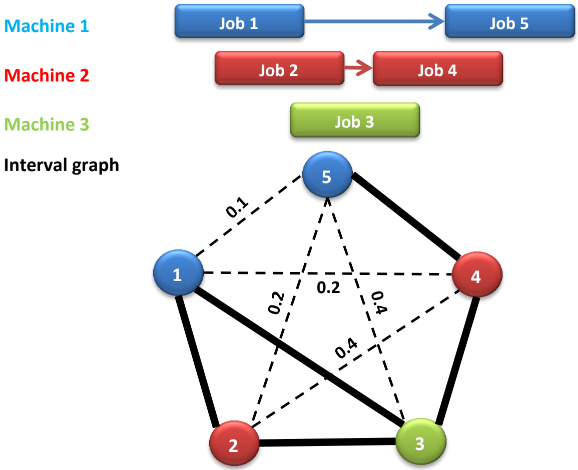
$$
\begin{aligned}
\min_{x,y} \mathbb{E}_{\xi} \left[\sum_{c \in \mathcal{C}} \sum_{j \in \mathcal{J}} y_{jc}(\xi) \right] & & \\
\sum_{j:\ s_j \le t < f_j^0} x_{jc} &\le 1,\ t \in \hat{\mathcal{T}},\ c \in \mathcal{C}, \\
\sum_{c \in \mathcal{C}} x_{jc} &= 1,\ j \in \mathcal{J}, \\
x_{jc} &\in \{0,1\},\ c \in \mathcal{C}, j \in \mathcal{J}, \\
\sum_{k:\ f_j^0 \le s_k < f_j(\xi)} x_{kc} &\le y_{jc}(\xi) + |\mathcal{J}|(1 - x_{jc}),\ c \in \mathcal{C}, j \in \mathcal{J}, \\
y_{jc}(\xi) &\in \mathbb{N},\ c \in \mathcal{C}.
\end{aligned}
\tag{2}
$$

$y_{jc}(\xi)$ express the number of jobs which cannot be processed by machine c due to the random delay in processing job j.

Contents

Relation to robust coloring problem

Feasible coloring with reliability (1-0.1)(1-0.4) $= 0.54$



Robust coloring problem
Notation

The interval graph for FIS:

- $\mathcal{C}$ available machines (colors),
- $\mathcal{J}$ set of vertices,
- set of (hard) edges E with overlapping pairs of jobs $\{j, j'\}$, i.e. $s_j \leq s_{j'} < f_j^0$.
- set of complementary (soft) edges $\overline{E}$ with all pairs $\{j, j'\}$ such that delay of job j can influence job j' if it is processed by the same machine, under unbounded support of $D_j(\xi)$ if $f_j^0 \leq s_{j'}$.

$$E \cap \overline{E} = \emptyset$$

ASSUMPTION: the number of available machines (colors) is greater or equal to the chromatic number of the graph $\mathcal{J}$, E.

Robust coloring problem

Yanez and Ramirez (2003):

$$\min_{x_{jc}, y_{jj'}} \sum_{\{j,j'\} \in \overline{E}} q_{jj'} y_{jj'}$$

$$\begin{aligned} \sum_{c \in \mathcal{C}} x_{jc} &= 1, \; j \in \mathcal{J}, \\ x_{jc} + x_{j'c} &\leq 1, \; \{j, j'\} \in E, c \in \mathcal{C}, \\ x_{jc} + x_{j'c} &\leq 1 + y_{jj'}, \; \{j, j'\} \in \overline{E}, c \in \mathcal{C}, \\ x_{jc} &\in \{0, 1\}, \; c \in \mathcal{C}, j \in \mathcal{J}, \\ y_{jj'} &\in \{0, 1\}, \; \{j, j'\} \in \overline{E}. \end{aligned} \tag{3}$$

- Penalty $q_{jj'}$ assigned to edges from $\overline{E}$ if the connected vertices share the same color,
- exactly one color is assigned to vertex j,
- forbids an identical coloring to hardly connected vertices,
- $y_{jj'} = 1$ for equally colored vertices connected in $\overline{E}$.

Schedule reliability maximization – independence

ASSUMPTION: The delays are mutually **independent**.

Penalties suggested by Yanez and Ramirez (2003):

$$q_{jj'} = -\ln\left(P(D_j(\xi) \leq s_{j'} - f_j^0)\right) = -\ln(p_{jj'}),$$

for $y_{jj'} = 1$, $\{j, j'\} \in \overline{E}$ obtained

$$\sum_{\{j,j'\}\in\overline{E}:\ y_{jj'}=1} q_{jj'} = -\ln\left(\prod_{\{j,j'\}\in\overline{E}:\ y_{jj'}=1} p_{jj'}\right),$$

which is equal to the minus logarithm of the whole schedule reliability in the case of independence of the machine overload. This formula does not hold for FISuU, a simple example ...

Simple example

Three jobs scheduled to the same machine, i.e. $s_1 < f_1^0 \leq s_2 < f_2^0 \leq s_3 < f_3^0$, and $y_{12} = y_{13} = y_{23} = 1$. Let the delays $D_1(\xi), D_2(\xi)$ be independent. Then the reliability of the schedule is equal to

$$
\begin{aligned}
& P(D_1(\xi) \leq s_2 - f_1^0, D_1(\xi) \leq s_3 - f_1^0, D_2(\xi) \leq s_3 - f_2^0) \\
= \; & P(D_1(\xi) \leq s_2 - f_1^0, D_2(\xi) \leq s_3 - f_2^0) \\
= \; & (1 - p_{12})(1 - p_{23}) \\
\neq \; & (1 - p_{12})(1 - p_{13})(1 - p_{23}).
\end{aligned}
$$

Schedule reliability maximization – copula dependence

We say that $C: [0,1]^J \to [0,1]$ is a J-**dimensional copula** if it satisfies

1. $C(u_1, \ldots, u_{i-1}, 0, u_{i+1}, \ldots, u_J) = 0$ for arbitrary i,
2. $C(1, \ldots, 1, u_i, 1, \ldots, 1) = u_i$ for arbitrary i,
3. C is J-increasing, i.e. for each $B = \prod_{i=1}^{J}[a_i, b_i] \subseteq [0,1]^J$

$$\int_B dC(u) = \sum_{z \in \times_{i=1}^{J}\{a_i, b_i\}} (-1)^{\mathrm{card}\{i:\ z_i = a_i\}} C(z) \geq 0.$$

Joint distribution of delays using their marginal distributions F_j

$$P(D_1(\xi) \leq d_1, \ldots, D_J(\xi) \leq d_J) = C(F_1(d_1), \ldots, F_J(d_J)).$$

If each d_j corresponds to the difference between the prescribed job end f_j^0 and a start of the subsequent job s_k, then we obtain the schedule reliability. Note that we set $d_j = \infty$ if there is no subsequent job.

Archimedean copula

A copula C is called **Archimedean** if there exists a continuous strictly decreasing function $\psi : [0,1] \to \mathbb{R}^+$ such that $\psi(1) = 0$ and

$$C(u) = \psi^{-1}\left(\sum_{i=1}^{J} \psi(u_i)\right),$$

where ψ is called the **generator** of the Archimedean copula C. These are the most important generators and corresponding copulas:

- The independent copula $\psi(u) = -\log(u)$,
- Clayton copulas $\psi(u) = \theta^{-1}(u^{-\theta} - 1)$, $\theta > 0$,
- Gumbel copulas $\psi(u) = (-\log(u))^{\theta}$, $\theta \geq 1$,
- Frank copulas $\psi(u) = -\log(\frac{e^{-\theta u} - 1}{e^{-\theta} - 1})$, $\theta > 0$.

Archimedean copula

Since the random delays are assumed to have **Archimedean copula dependence**, **the schedule reliability** R of coloring x can be expressed as

$$\begin{aligned} R(x) &= P\left(\xi \in \Xi : \sum_{j:\ s_j \le t < f_j(\xi)} x_{jc} \le 1,\ t \in \hat{\mathcal{T}}, c \in \mathcal{C}\right) \\ &= \psi^{-1}\left(\sum_{j=1}^{J} \psi(u_j)\right), \end{aligned}$$

with $u_j = P\left(D_j(\xi) \le \min_{k:\ y_{jk}=1\ \&\ s_k \ge f_j^0} s_k - f_j^0\right)$, where we are using the convention for the minimum over an empty set $\min_\emptyset = +\infty$, i.e. $P\left(D_j \le \min_\emptyset s_k - f_j^0\right) = 1$ if job j has no successors scheduled to the same machine.

$$q_{jj'} = \psi\left(P(D_j(\xi) \le s_{j'} - f_j^0)\right),\ \{j, j'\} \in \overline{E}.$$

Extended robust coloring problem

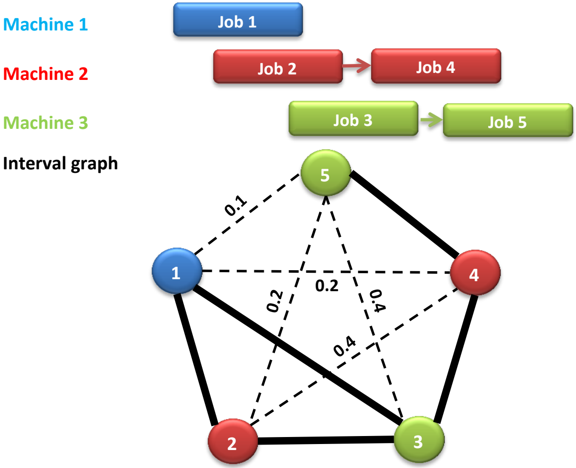
ASS.: Jobs sorted according their starting times s_j (no ties).

$$
\begin{aligned}
\min_{x,y,z} \sum_{\{j,j'\}\in\overline{E}} q_{jj'} z_{jj'} \quad \text{s.t.} \quad \sum_{c\in\mathcal{C}} x_{jc} &= 1,\ j\in\mathcal{J},\\
x_{jc} + x_{j'c} &\leq 1,\ \{j,j'\}\in E,\\
x_{jc} + x_{j'c} &\leq 1 + y_{jj'},\ \{j,j'\}\in\overline{E},\\
y_{jj'} + \sum_{k:\ \{j,k\}\in\overline{E}\ \&\ s_k\geq f^0_{j'}} z_{jk} &\leq 1,\ \{j,j'\}\in\overline{E},\\
\sum_{k:\{j,k\}\in\overline{E}} y_{jk} &\leq |\mathcal{J}|\cdot\sum_{k:\{j,k\}\in\overline{E}} z_{jk},\ j\in\mathcal{J},\\
x_{jc}, y_{jk}, z_{jk} &\in \{0,1\},\ c\in\mathcal{C}, j,k\in\mathcal{J},
\end{aligned}
\tag{4}
$$

where $z_{jk}=1$ if job k is a successor of j and share the same color.

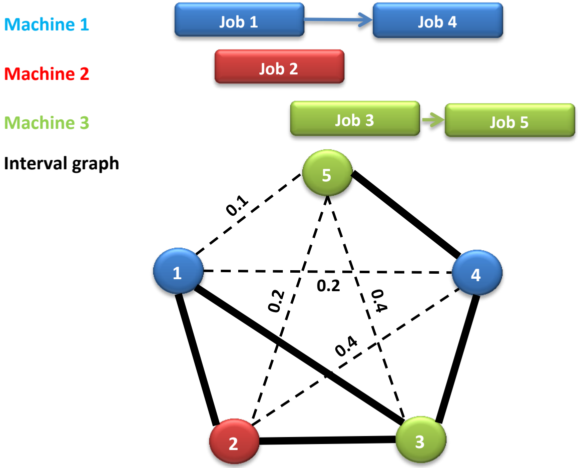
Relation to robust coloring problem

Feasible coloring with reliability (1-0.4)(1-0.4) $=$ 0.36



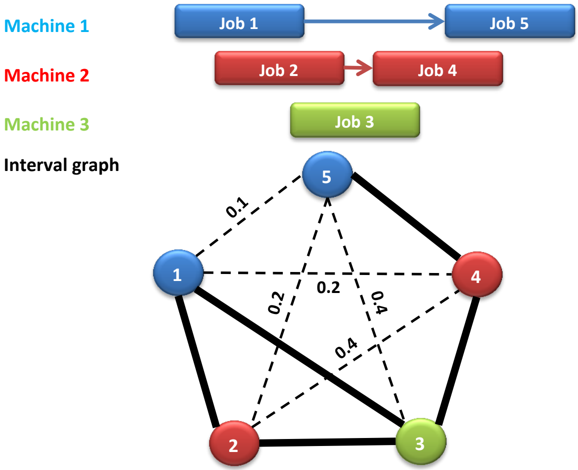
Relation to robust coloring problem

Feasible coloring with reliability (1-0.2)(1-0.4) $= 0.48$



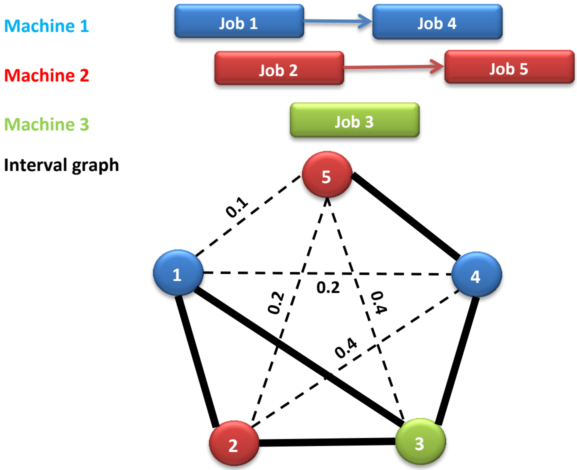
Relation to robust coloring problem

Feasible coloring with reliability (1-0.1)(1-0.4) = 0.54



Relation to robust coloring problem

Feasible coloring with reliability (1-0.2)(1-0.2) $= 0.64$



Contents

Simulated test instances

We consider 40 original test instances, which were simulated using the exponential distributions for the job processing times (with parameter λ_1) and breaks between jobs (λ_2):

- Instances 1–10: 30 jobs assigned to 5 machines, $\lambda_1 = 0.2$, $\lambda_2 = 0.05$ (simulated on 5 machines with 6 jobs)
- 11–20: 30 jobs, 5 machines, $\lambda_1 = 0.1$, $\lambda_2 = 0.05$ (5m, 6j)
- 21–25: 100 jobs, 10 machines, $\lambda_1 = 0.2$, $\lambda_2 = 0.05$ (5m, 20j)
- 25–30: 100 jobs, 10 machines, $\lambda_1 = 0.2$, $\lambda_2 = 0.1$ (5m, 20j)
- 31–35: 250 jobs, 20 machines, $\lambda_1 = 0.2$, $\lambda_2 = 0.05$ (10m, 25j)
- 35–40: 250 jobs, 30 machines, $\lambda_1 = 0.2$, $\lambda_2 = 0.05$ (25m, 10j)

Simulated test instances

Solution:

- CPLEX 12.1 solver available in the modeling system GAMS 23.2.
- PC with Intel Core i7 2.90 GHz CPU, 8 GB RAM and 64-bit Windows 7 Professional operational system
- Time limit set to 1 hour.
- Matlab implementation of tabu search (parameters: No of iterations, maximal length of tenures, random neighbourhood).

Tabu search

1. INITIALIZATION:
   I. Find a feasible coloring using the left edge algorithm for the graph $(\mathcal{J}, E)$.
   II. Set the tabu lists (tabu1, tabu2) to zero.
   III. Set the actual $\tilde{x}_0$ and the best solution $\hat{x}$ equal to the initial solution.
2. THE MAIN CYCLE: For i = 1 to MaxIterations do:
   I. Find the best solution y in a neighbourhood of the actual solution $\tilde{x}_{i-1}$ – For all vertices which are allowed (tabu1) to change the color do:
      i. Assign a new allowed (tabu2) color to the vertex
      ii. Compute the reliability – *if the solution* $\hat{x}$ *has not changed for 20 iterations, then allow infeasible solutions*
      iii. If the reliability is higher than the best one already found, save the coloring to y
   II. Set the actual solution to y, i.e. $\tilde{x}_i = y$, and actualize the tabu lists (vertex, new color) by random tenures.
   III. If the actual coloring $\tilde{x}_i$ has a higher reliability, i.e. $R(\tilde{x}_i) > R(\hat{x})$, then actualize the best solution and set $\hat{x} = \tilde{x}_i$.

Instances with 30 jobs – results of GAMS (CPlex) and tabu search algorithm (MatLab)

GAMS/ TS	Test instance									
	11	12	13	14	15	16	17	18	19	20
Reliability	94.9%	83.4%	91.5%	65.4%	90.1%	96.1%	65.7%	88.7%	80.6%	82.2%
Time	2:00	LIMIT	14:26	LIMIT	49:40	3:36	11:23	LIMIT	LIMIT	LIMIT
Avg. rel.	94.6%	82.5%	91.2%	63.7%	89.3%	95.3%	63.7%	88.5%	79.6%	81.7%
Min. rel.	94.6%	82.2%	90.8%	63.5%	89.2%	95.0%	63.2%	88.5%	79.6%	81.7%
Max. rel.	94.7%	83.0%	91.4%	64.2%	89.7%	95.7%	64.0%	88.5%	79.7%	81.8%
Abs. diff. (Avg)	-0.3%	-0.9%	-0.3%	-1.6%	-0.8%	-0.8%	-2.0%	-0.3%	-1.0%	-0.5%
Abs. diff. (Min)	-0.3%	-1.2%	-0.7%	-1.9%	-1.0%	-1.1%	-2.6%	-0.3%	-1.0%	-0.5%
Abs. diff. (Max)	-0.2%	-0.4%	-0.1%	-1.1%	-0.4%	-0.4%	-1.7%	-0.2%	-0.9%	-0.4%
Rel. diff. (Avg)	-0.3%	-1.1%	-0.3%	-2.5%	-0.9%	-0.8%	-3.1%	-0.3%	-1.2%	-0.6%
Rel. diff. (Min)	-0.3%	-1.5%	-0.8%	-2.8%	-1.1%	-1.1%	-3.9%	-0.3%	-1.2%	-0.6%
Rel. diff. (Max)	-0.2%	-0.5%	-0.1%	-1.7%	-0.4%	-0.4%	-2.7%	-0.3%	-1.1%	-0.5%
Avg. time	1:17	1:15	1:21	1:14	1:17	1:9	1:20	1:13	1:14	1:14

Instances with 100 jobs – average reliabilities (10 runs)

Max. tenures	Iter.	Test instance 21	22	23	24	25	26	27	28	29	30
TS 60-120	0	0.7%	0.7%	0.5%	1.0%	0.6%	0.4%	0.8%	1.0%	0.5%	0.7%
	1000	78.6%	72.8%	68.3%	72.7%	76.8%	84.2%	84.7%	87.8%	89.0%	90.6%
	2000	78.9%	73.1%	68.4%	73.4%	77.4%	84.3%	85.0%	88.3%	89.3%	91.0%
	3000	79.0%	73.2%	68.5%	73.6%	77.8%	84.5%	85.1%	88.6%	89.6%	91.1%
	4000	79.0%	73.3%	68.6%	73.7%	78.0%	84.6%	85.2%	88.7%	89.7%	91.2%
	5000	79.0%	**73.3%**	68.7%	**73.9%**	**78.2%**	84.7%	85.2%	**88.8%**	**89.9%**	**91.3%**
Avg. time		24:34	24:29	25:19	24:54	24:46	25:45	25:12	25:23	25:11	25:11
TS 60-120 Rand. sel. 30%	0	0.7%	0.7%	0.5%	1.0%	0.6%	0.6%	0.8%	1.0%	0.5%	0.7%
	1000	77.5%	72.6%	69.3%	72.3%	76.9%	84.3%	85.1%	87.7%	88.8%	90.3%
	2000	77.9%	72.8%	69.8%	72.6%	77.5%	84.6%	85.3%	88.1%	89.1%	90.8%
	3000	78.2%	73.2%	70.0%	72.9%	77.7%	84.7%	85.4%	88.3%	89.3%	91.0%
	4000	78.3%	73.2%	70.3%	73.1%	78.0%	84.7%	85.5%	88.3%	89.4%	91.0%
	5000	78.3%	73.2%	**70.4%**	73.3%	78.0%	**84.9%**	**85.5%**	88.5%	89.5%	91.0%
Avg. time		7:23	7:15	7:17	7:12	7:20	7:17	7:17	7:15	7:13	7:17

Contents

Flow-based formulation

We define penalties

$$q_{jkc} = \psi\left(P(D_{jc}(\xi) \leq s_k - f_j)\right),$$

Two artificial jobs 0, $J+1$ (= machine start and end):

$$\begin{aligned} \mathcal{J}_0 &= \mathcal{J} \cup \{0\}, \\ \mathcal{J}_{J+1} &= \mathcal{J} \cup \{J+1\}, \\ \mathcal{J}_{0(J+1)} &= \mathcal{J} \cup \{0\} \cup \{J+1\}. \end{aligned} \tag{5}$$

We extend the set of edges $\bar{E}$ by $\{0, J+1\}$, $\{0, j\}$, $\{j, J+1\}$, $\forall j \in \mathcal{J}$ with penalties $q_{0(J+1)c} = q_{0jc} = q_{j(J+1)c} \equiv 0$. We denote by $\bar{E}_{.j}$ the set of possible predecessors of job $j \in \mathcal{J}$, i.e.

$$\bar{E}_{.j} = \{j' \in \mathcal{J}_0 : \{j', j\} \in \bar{E}\},$$

and by $\bar{E}_{j.}$ the set of allowed successors

$$\bar{E}_{j.} = \{j' \in \mathcal{J}_{J+1} : \{j, j'\} \in \bar{E}\}.$$

Flow-based formulation

Branda and Hájek (2016):

$$
\begin{aligned}
\min_{y} \quad & \sum_{c\in\mathcal{C}} \sum_{\{j,j'\}\in\bar{E}} q_{jj'c} y_{jj'c} \\
\text{s.t.} \quad & \sum_{j\in\mathcal{J}_{J+1}} y_{0jc} = 1, \quad c\in\mathcal{C}, \\
& \sum_{k\in\bar{E}_{\cdot j}} y_{kjc} = \sum_{j'\in\bar{E}_{j\cdot}} y_{jj'c}, \quad j\in\mathcal{J}, \quad c\in\mathcal{C}, \\
& \sum_{j\in\mathcal{J}_0} y_{j(J+1)c} = 1, \quad c\in\mathcal{C}, \\
& \sum_{c\in\mathcal{C}} \sum_{j'\in\bar{E}_{j\cdot}} y_{jj'c} = 1, \quad j\in\mathcal{J}, \\
& y_{jj'c}\in\{0,1\}, \quad j,j'\in\mathcal{J}, \quad c\in\mathcal{C}.
\end{aligned}
\tag{6}
$$

Contents

Parameters:

1. number of jobs J,
2. number of machines C,
3. λ_1 – rate parameter of the exponential distribution – starting time of a job,
4. λ_2 – rate parameter of the exp. d. – length of the processing interval,
5. p – probability that no delay appears, i.e. a job is finished in time,
6. λ_3 – rate parameter of the exp. d. – length of the random delay if it appears with probability $(1-p)$; it enables to compute the coefficients $q_{jj'c} = q_{jj'}$, independent delays $\psi(u) = -\log u$.

Input parameters 1:

$$\lambda_1 = \frac{1}{2}, \quad \lambda_2 = \frac{1}{5}, \quad \lambda_3 = \frac{1}{2,5}, \quad p = \frac{1}{3},$$

Input parameters 2:

$$\lambda_1 = \frac{1}{2.5}, \quad \lambda_2 = \frac{1}{5}, \quad \lambda_3 = \frac{1}{2}, \quad p = \frac{1}{2}.$$

Problem size		Average computational time [s]			
		Input parameters 1		Input parameters 2	
No. of jobs	No. of machines	FBF	ERCP	FBF	ERCP
10	5	0.12	0.48	0.13	0.54
13	6	0.18	3.24	0.18	3.33
15	6	0.19	46.05	0.19	30.55
17	7	0.20	66.42	0.21	46.80
20	8	0.31	254.24	0.27	328.30
22	8	0.34	652.77	0.38	457.05
25	9	0.43	707.31	0.42	664.63
27	10	0.66	920.14	0.67	886.66
30	11	0.74	879.97	0.75	837.92
35	12	0.94	928.72	0.97	918.24
40	15	1.74	1000.14	1.48	1000.13

Larger instances in Branda and Hájek (2016) ..

Contents

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