Convex Optimization 2025/26

Practical session # 10

December 4, 2025

1. Find the maximum likelihood (ML) estimate for linear measurements, where the noise is exponentially distributed with density

$$p(z) = \begin{cases} \frac{1}{a}e^{-z/a} & z \geq 0 \\ 0 & z < 0, \end{cases}$$

where $a > 0$.

2. Vilfredo Pareto (you already know him from *Pareto optima*) studied wealth distributions in different countries. He observed that they can usually be modeled by power-law distributions (also called *Pareto distributions*). Such distributions are given by the density function

$$p(x) = \begin{cases} \frac{\alpha x_m^\alpha}{x^\alpha} & x \geq x_m \\ 0 & x < x_m, \end{cases}$$

where $x_m > 0$ and $\alpha > 0$ are parameters.

- Assume you have a random data sample $u_1, u_2, \ldots, u_n \in \mathbf{R}$ of the personal wealth of citizens of Prague (and fixed lower bound $x_m > 0$). Determine the ML estimate for the parameter α.
- Can we also get an ML estimate for x_m?

3. (Probit model) Suppose $y \in \{0, 1\}$ is a random variable given by

$$y = \begin{cases} 1 & a^T u + b + v \leq 0 \\ 0 & a^T u + b + v > 0, \end{cases}$$

where $u \in \mathbf{R}^n$ is a vector of explanatory variables, and v describes some noise following a standard normal distribution. Formulate the ML estimation problem for estimating $a \in \mathbf{R}^n$ and $b \in \mathbf{R}$, given data pairs $(u_i, y_i) \in \mathbf{R}^n \times \{0, 1\}$ for $i = 1, \ldots, m$. You don't need to solve it.

Is this a convex optimization problem? To solve this question, derive first a second-order criterion for $\log(f(x))$ to be concave, for some $f\colon \mathbf{R} \to \mathbf{R}$.

4. Let us consider a general penalty function approximation (minimize $\sum_{i=1}^m \phi(b_i - a_i^T x)$), for a penalty function ϕ. Try to find a noise distribution such that this is equivalent to the ML estimation problem for linear measurements (a_i, b_i) for $i = 1, \ldots, m$.